LIMITATIONS OF THE STANDARD INTERACTION FORMULA FOR BIAXIAL BENDING AS APPLIED TO RECTANGULAR STEEL TUBULAR COLUMNS

Ramon V. Jarquio[1]

ABSTRACT

The limitations of the standard interaction formula for biaxial bending can be shown by the results of the analytical method in structural analysis. The analytical method utilizes the known stress/strain properties of the materials and the geometry of the rectangular tubular section. With this method it is possible to solve for the capacity of the rectangular section for biaxial bending at any position of its capacity axis from the horizontal to the vertical.

In contrast, the standard flexure and interaction formula determine capacities at the horizontal and vertical positions only and thus incapable of directly solving the capacity for biaxial bending.

The tabulations will show that the standard interaction formula for biaxial bending can only capture about 50% of capacity while the analytical method can capture 100%. Moreover, biaxial bending capacity is only about 80% of the uniaxial capacity in steel manuals and hence may be risky to use in design practice.

This paper will alert structural engineers that the analytical method is a more accurate tool to use than the standard interaction formula for biaxial bending. The availability of computers can easily handle the solution of hundreds of equations required to solve the capacity of a given rectangular section.

KEY WORDS

analytical method, capacity curve, design computation and guideline, standard interaction formula

INTRODUCTION

Steel Tubular columns can be hollow or concrete-filled. For concrete-filled steel tubular section, the concrete core is allowed to develop its ultimate strength. In this condition of stress/strain in the concrete, the steel shell reaches the yield stress since steel strain is less than that of the concrete. For hollow section the outermost part of the shell from the neutral axis is allowed to reach yield stress. In this presentation we will assume this condition prevails.

To implement the analytical method, all variable parameters should be accounted for in the analysis. These variables are the dimensions of the rectangular steel tubular section and the ultimate and yield stress of concrete and steel respectively. The rectangular steel tubular

[1] Retired Director of Engineering Support, New York City Transit, 35-23 208th St., Bayside, New York 11361, Phone 718/229-3408, FAX 718/229-5212, rvjarquio@aol.com

section can then be drawn and these parameters labeled. In addition a line is drawn through the center of the section with an inclination anywhere between the horizontal axis and the vertical axis. This line is designated as the capacity axis. For practical purposes, this line should lie along the diagonal of the rectangular section as a limit since most resultant external load will lie within the sector defined by the horizontal axis and the diagonal. The line perpendicular to this capacity axis will designated as the moment axis. The values of bending moments obtained will be parallel to the capacity axis and represents the component of the resultant moment. To obtain the resultant moment capacity of the section, moments about the capacity axis is also calculated. The square root of the sum of the squares of these moments is the value of the resultant bending moment for biaxial bending capacity of the section.

It will require hundreds of equations to complete the expressions for the forces and moments which the section can develop at its ultimate strength conditions. Most of these equations have been published and listed in papers presented by the author in several international conferences in structural engineering and construction. However, the basic formulations in the derivations will be shown to enable the reader to verify the published equations if he or she chooses to do so. There are no other references to cite except the author's published works since standard literature uses the traditional interaction formula for biaxial bending. It is only by using the results of the analytical method that a comparison can be made to expose the crudeness of the current one-line equation standard interaction formula for biaxial bending.

DERIVATION

HOLLOW STEELTUBULAR SECTION

Figure 1 shows a rectangular steel tubular section with dimensions b and d. The thickness of the shell is designated as t and the column capacity axis as the X-axis and the moment axis as the Z-axis. The inclination of the capacity axis is designated as θ from the horizontal. Draw lines perpendicular to the X-axis and passing through the corners and the centre of the outer rectangle. These will divide the section into three main stress volumes designated as V_1, V_2 and V_3 and their corresponding bending moments V_1x_1, V_2x_2 and V_3x_3. Designate the sides of the rectangle as lines z_1, z_2, z_3 and z_4. Designate the ordinate of the outer rectangle as z_m and the abscissa as h/2. Using the point-slope form of the straight line from analytic geometry, write the equations of the sides of the rectangle.

In the XY plane draw the stress/strain for steel. The equation of the steel stress/strain diagram is that of a straight line also and can be easily written as above. Draw this line for a typical compressive depth and designate its distance from the compressive edge as c.

The intersections of the steel stress diagram with the lines from the corners and centre of the rectangular section will define the limits of the different boundaries of the stress volumes and thus the forces and moments to be calculated. For this column section there will be at least five ranges of values for c to be calculated.

Write the first set of equations for the outer rectangular section and then the inner rectangular section will have the same number of equations. There are 96 equations to write for the solution of the yield capacity curve of the rectangular steel tubing (Jarquio 2005). The steel forces and moments for the steel tubing are obtained by subtracting the values of the inner rectangular section from the outer rectangular section. Some basic formulations before the integration of the stress volumes and their corresponding bending moments can be performed are as follows:

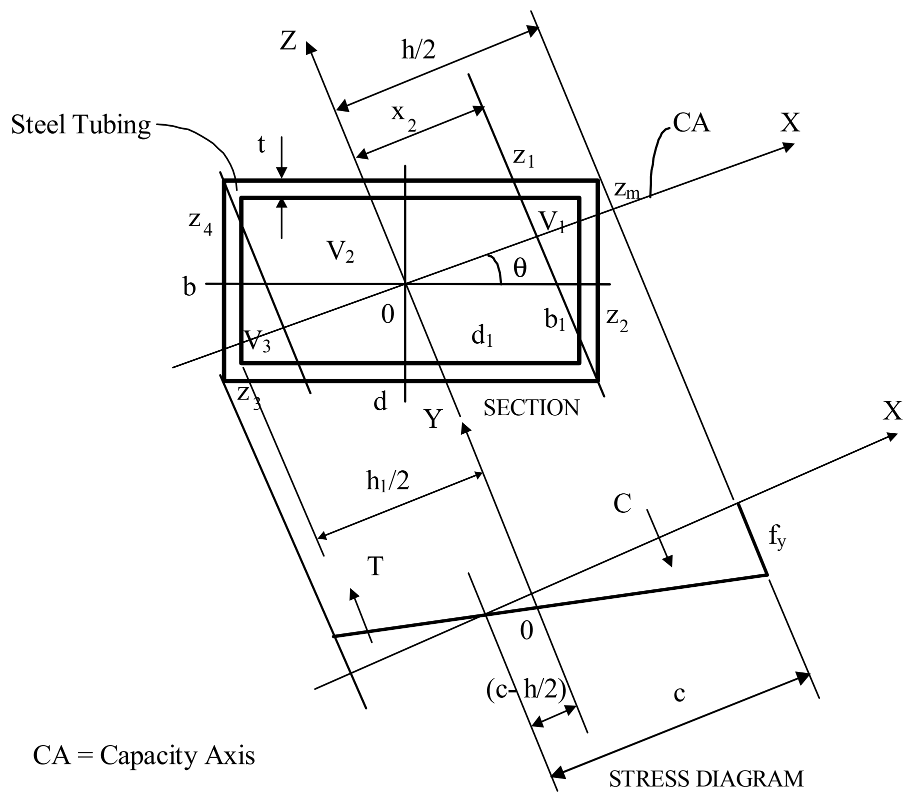


Figure 1: Rectangular Steel Tubing

The equations of the lines of the outer rectangle are represented by the following expressions:

$z_1 = -\tan\theta\,(x - h/2) + z_m$ (1)

$z_2 = \cot\theta\,(x - h/2) + z_m$ (2)

$z_3 = -\tan\theta\,(x + h/2) - z_m$ (3)

$z_4 = \cot\theta\,(x + h/2) - z_m$ (4)

The equation of the stress diagram is

$y = (f_y/c)[x + (c - h/2)]$ (5)

$x_2 = (1/2)(d\sin\theta - b\cos\theta)$ (6)

$h = b\cos\theta + d\sin\theta$ (7)

$z_m = (1/2)(d \cos \theta - b \sin \theta)$ (8)

With the above basic equations to start the derivation, we can proceed to use integral calculus to derive the forces and moments which can be developed in a rectangular section. These equations represent the solution for the capacity of any rectangular steel tubing. These equations are set up in Excel spreadsheets to generate the values at specific position of the compressive depth c at any chosen position of the capacity axis. For square tubing use b = d in the above derived equations to obtain the yield capacity curve for this case.

CONCRETE-FILLED STEEL TUBULAR COLUMN

Figure 2 is a concrete-filled rectangular steel tubular column with similar notations except for the stress volume notations plus the concrete core inside the tube. Here, the stress diagram is that of the CRSI (Concrete Reinforcing Steel Institute), wherein the compressive and tensile steel stresses are mostly in yield conditions when the compressive depth of concrete c is less than that at balanced conditions. This is different from the author's published solution in his book (Jarquio, 2004) in which the tensile stress at the farthest point from the neutral axis is held at the value of the yield stress and decreases as the compressive depth is decreased from balanced condition.

As before, we draw lines perpendicular to the X-axis from the corners and center of the rectangular section. This is done to delineate the stress volumes V_1, V_2 and V_3 together with their corresponding bending moments V_1x_1, V_2x_2 and V_3x_3 to be calculated. There will be twice as much number of ranges of values for the compressive depth c as was in the hollow steel case above.

We can use the same initial equations as before in the integration of steel forces and moments. First determine the equations for the steel forces and moments applicable to the outer rectangular section. This procedure will yield the steel forces and bending moments which can be developed at the ultimate condition of the concrete core. The limiting concrete strain = 0.003 is the reference for corresponding steel strains in the column section. The steel forces and moments are then computed. The same procedure is followed for the inner rectangular steel section.

This ultimate strength capacity is represented by a curve of plotted values from the Excel spreadsheets. The vertical scale is for the axial capacity, P and the horizontal scale is for the bending moment capacity, M of the column section. The eccentricity, e = M/P represents the displacement of the column section with respect to the plumb line due to buckling of the column length. When buckling is not present as in a stub column, the eccentricity represents the equivalent position of the axial load from the plumb line.

The capacity curve show the dependence of the axial capacity with the bending moment, i.e., when the bending moment is increased, the axial capacity is decreased. On the other hand, when the axial capacity is increased, the bending moment is decreased. The accidental eccentricity due to out of plumbness of the column section itself, plus deviation of the application of the external axial and bending moment loads at supports will add external bending moment acting on the section. When this occurs, the internal capacity of the section, i.e. the axial capacity should also be reduced. For this reason, the final external axial and bending moment loads determined by the structural engineer are the pair of values to be plotted on the capacity curve to determine adequacy of the given section to support these loads.

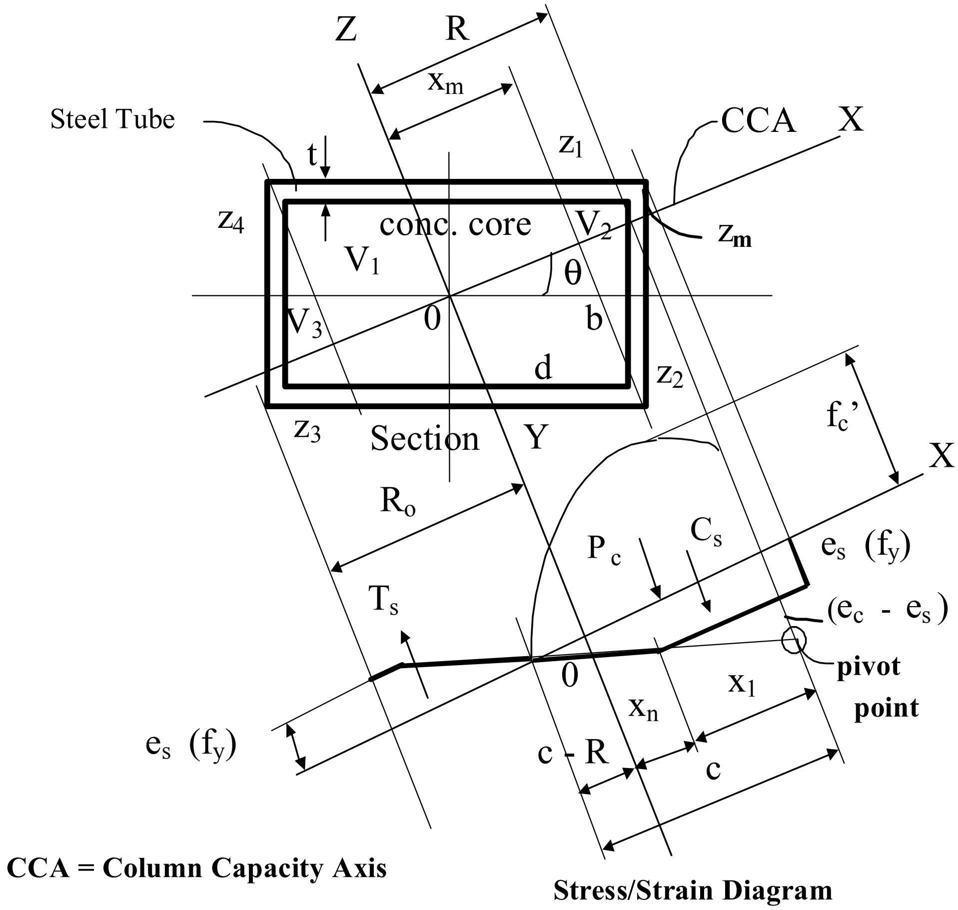


Figure 2: Rectangular CFT Steel Column

CAPACITY CURVES

For any position of the capacity axis from horizontal to the vertical position, a capacity curve is generated that is unique to this particular position. Values of the bending moments will decrease from the horizontal position to the vertical position of the capacity axis. For a rectangular section, the diagonal is chosen as the axis to use for biaxial bending since most of the external resultant load will fall within the sector defined by the horizontal and the diagonal.

This capability of the analytical method can not be matched by the standard interaction formula for biaxial bending. The analytical method can determine the centroid of the internal capacity of the section at every particular position of the compressive depth c of concrete. A structural engineer can demonstrate the equilibrium conditions of external and internal loads by the analytical method while the standard interaction formula has no such capability.

Example 1: Plot the capacity curve along the diagonal of a hollow rectangular steel tubular column 203.2 mm.(8 inches) x 304.8 mm.(12 inches) with shell thickness 12.7 mm. (0.50 inch) and f_y = 248 MPa (36 kips per square inch). Figure 3 is the plot of the capacity curve for this example steel tubular column.

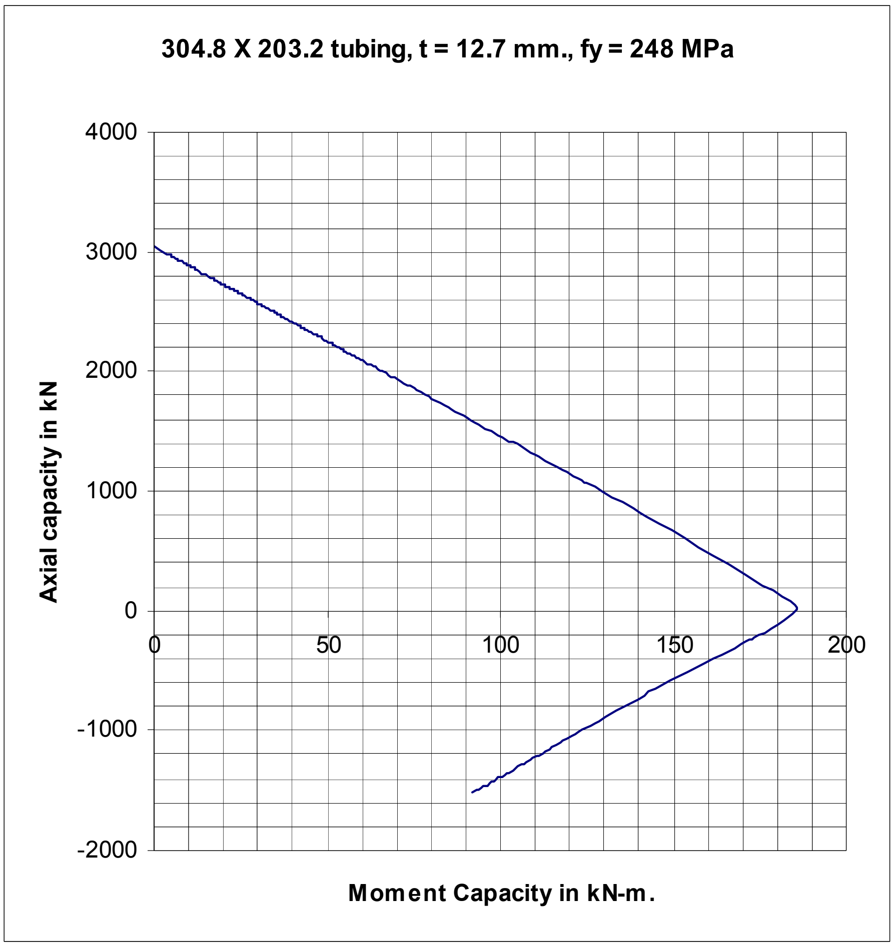
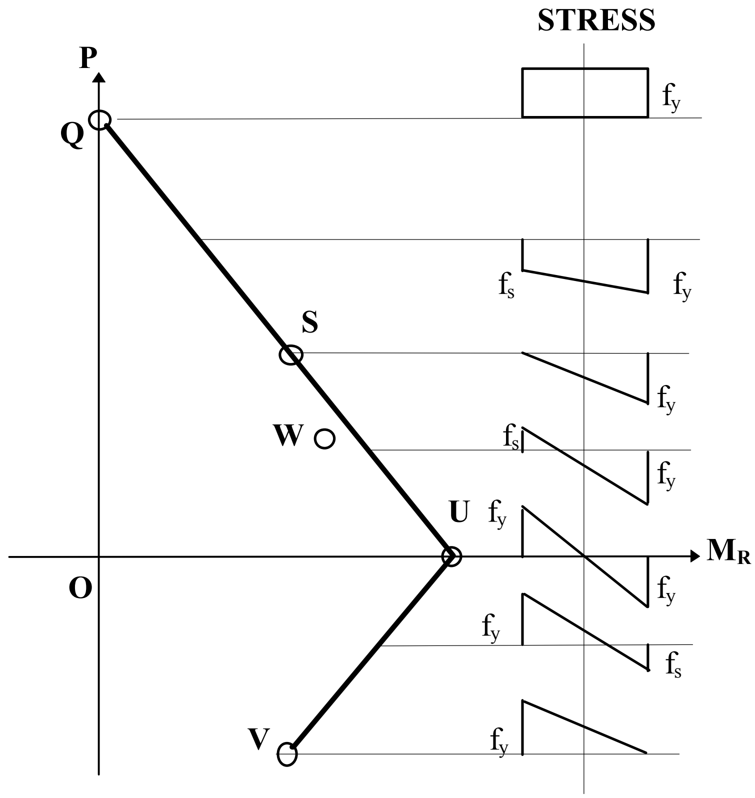


Figure 3: Capacity Curve of a Hollow Rectangular Steel Tubular Column

Figure 4 is the schematic location of the key points with their stress conditions in the capacity curve. W_o inside the envelope of the capacity curve represents a plot of a set of external load consisting of a pair of axial and bending moment. The selected section in this case is adequate to support the external load.

The part of the curve that is useful in structural analysis is the portion above the horizontal line at zero value of axial capacity. This could be either compression or tensile capacity as the case maybe since steel material can develop either strength equally. The lower part of the curve indicates tension capacity when part of the section is still subjected to compressive stresses. This condition of tension in the section occurs when the compressive depth c is less than that at balanced condition.



KEY POINT	c inches	MR in-kips	P kips	c cm.	MR kN-m.	P kN
		theta =	0.5880			
V	0.01	815	-342	0.03	92	-1521
U	7.20	1660	-1	18.29	188	0
S	14.42	851	342	6.60	96	1521
Q	c very large	0	684	c very large	0	3042

Figure 4: Schematic Locations of Key Points with Indicated Values

Example 2: Plot the capacity curve along the diagonal of a concrete-filled rectangular steel tubular column with the concrete core dimension b = 254 mm. (10 inches) and d =

355.6 mm. (14 inches) with shell thickness = 25.4 mm. (one inch) and f_c' = 32 MPa (5 kips per square inch), f_y = 345 MPa (50 kips per square inch). Figure 5 is the plot of the capacity curve from Excel spreadsheets with indicated values of the axial capacity which can be developed as a function of the compressive depth c.

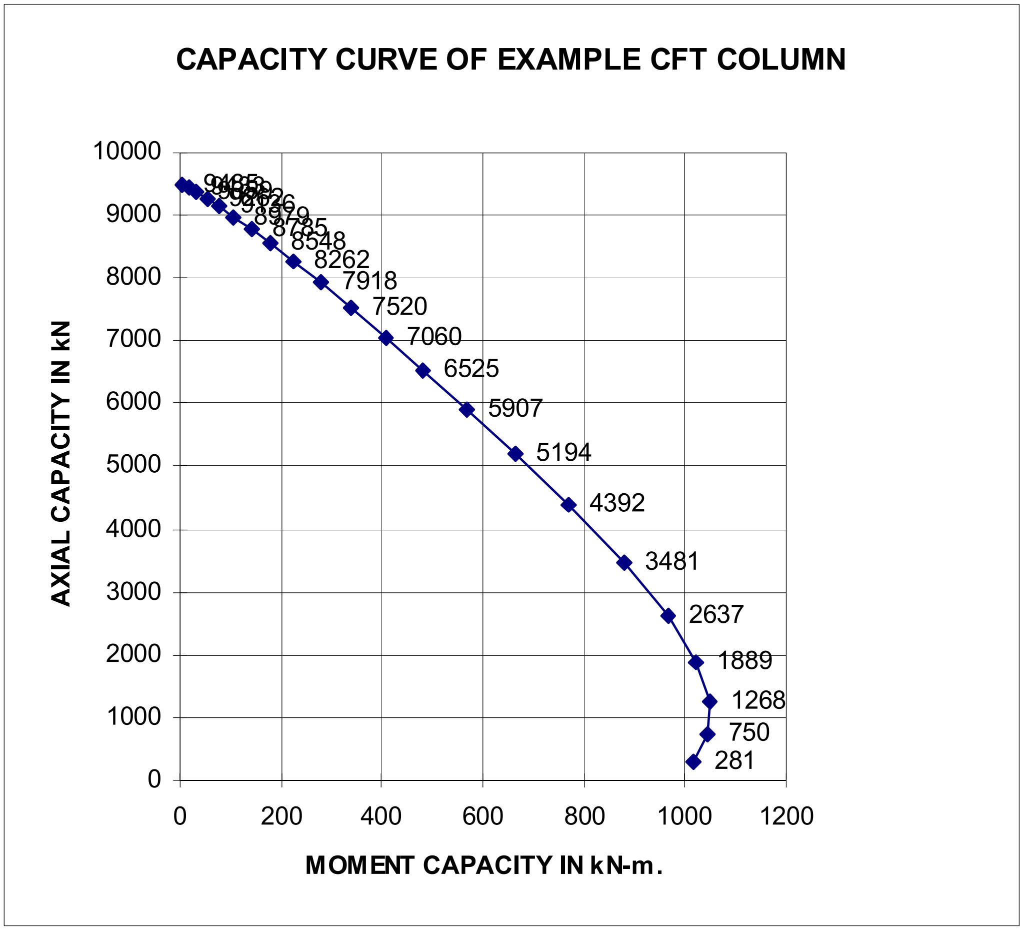


Figure 5: Plot of the Capacity Curve of the CFT Column

COMPARISON

Comparison between the analytical method and the standard interaction formula for biaxial bending can be made by using the results obtained above. The standard interaction formula for biaxial bending uses the sum of the ratios of the moments at orthogonal axes not to exceed unity or one.

Table 1 will show the results of example 1 and apply the standard interaction formula using the values as follows: At key point S, M_R = 96 kN-m., P = 1521 kN, θ = 0.588 radian, M_1 = 127 kN-m., M_2 = 100 kN-m., P_{max} = 3042 kN.

The table is constructed such that $M_1' = M_R' \cos\theta$; $M_2' = M_R' \sin\theta$; $R_1 = M_1'/M_1$; $R_2 = M_2'/M_2$; $R = M_R'/M_R$.

Table 1: Limitations of the Standard Interaction Formula for Biaxial Bending for Steel Tubing

M_1'	M_2'	M_R'	R_1	R_2	$R_1 + R_2$	R	%
80	53	96	0.629	0.533	1.161	1.00	100
78	52	94	0.616	0.521	1.137	0.98	98
77	51	92	0.603	0.510	1.113	0.96	96
75	50	90	0.590	0.499	1.089	0.94	94
73	49	88	0.577	0.488	1.065	0.92	92
72	48	86	0.563	0.477	1.040	0.90	90
70	47	84	0.550	0.466	1.016	0.88	88
68	**45**	**82**	**0.537**	**0.455**	**0.992**	**0.85**	**85**
67	44	80	0.524	0.444	0.968	0.83	83
65	43	78	0.511	0.433	0.944	0.81	81
63	42	76	0.498	0.422	0.919	0.79	79
62	41	74	0.485	0.410	0.895	0.77	77
60	40	72	0.472	0.399	0.871	0.75	75
58	39	70	0.459	0.388	0.847	0.73	73
57	38	68	0.446	0.377	0.823	0.71	71
55	37	66	0.432	0.366	0.799	0.69	69
53	36	64	0.419	0.355	0.774	0.67	67
52	34	62	0.406	0.344	0.750	0.65	65
50	33	60	0.393	0.333	0.726	0.63	63
48	32	58	0.380	0.322	0.702	0.60	60
47	31	56	0.367	0.311	0.678	0.58	58
45	30	54	0.354	0.300	0.653	0.56	56
43	29	52	0.341	0.288	0.629	0.54	54
42	28	50	0.328	0.277	0.605	0.52	52
40	27	48	0.314	0.266	0.581	0.50	50
38	26	46	0.301	0.255	0.557	0.48	48
37	24	44	0.288	0.244	0.532	0.46	46
35	**23**	**42**	**0.275**	**0.233**	**0.508**	**0.44**	**44**
33	22	40	0.262	0.222	0.484	0.42	42
32	21	38	0.249	0.211	0.460	0.40	40
30	20	36	0.236	0.200	0.436	0.38	38

Note that the sum of the ratios for biaxial bending can capture 85% of the capacity of the section. The ratio of the axial loads is equal to 1521/3042 = 0.50. When this ratio is added to $R_1 + R_2$ we have to go down under this column to obtain about only 44% of capacity. Also, the biaxial bending capacity is (96/127) x100% = 76 % of the uniaxial capacity.

Similarly For example 2 we have the analytical results as follows: At key point U, M_R = 1047 kN-m., P = 1268 kN, M_1 = 1229 kN-m., M_2 = 993 kN-m and P_{max} = 9485 kN. θ = 0.62025 radian. These values are used to construct Table 2 below.

Table 2: Limitations of the Standard Interaction Formula for Biaxial Bending for CFT Rectangular Column

M_1'	M_2'	M_R'	R_1	R_2	$R_1 + R_2$	R_3	$R_1 + R_2 + R_3$	R	%
852	609	1047	0.693	0.613	1.306	0.134	1.440	1.00	100
834	596	1025	0.679	0.600	1.279	0.134	1.412	0.98	98
814	581	1000	0.662	0.585	1.247	0.134	1.381	0.96	96
793	567	975	0.646	0.571	1.216	0.134	1.350	0.93	93
773	552	950	0.629	0.556	1.185	0.134	1.319	0.91	91
753	538	925	0.612	0.541	1.154	0.134	1.288	0.88	88
732	523	900	0.596	0.527	1.123	0.134	1.256	0.86	86
712	509	875	0.579	0.512	1.092	0.134	1.225	0.84	84
692	494	850	0.563	0.498	1.060	0.134	1.194	0.81	81
671	480	825	0.546	0.483	1.029	0.134	1.163	0.79	79
651	465	800	0.530	0.468	0.998	0.134	1.132	0.76	76
631	450	775	0.513	0.454	0.967	0.134	1.100	0.74	74
610	436	750	0.497	0.439	0.936	0.134	1.069	0.72	72
590	421	725	0.480	0.424	0.904	0.134	1.038	0.69	69
570	407	700	0.463	0.410	0.873	0.134	1.007	0.67	67
549	**392**	**675**	**0.447**	**0.395**	**0.842**	**0.134**	**0.976**	**0.64**	**64**
529	378	650	0.430	0.380	0.811	0.134	0.945	0.62	62
509	363	625	0.414	0.366	0.780	0.134	0.913	0.60	60
488	349	600	0.397	0.351	0.748	0.134	0.882	0.57	57
468	334	575	0.381	0.337	0.717	0.134	0.851	0.55	55
448	320	550	0.364	0.322	0.686	0.134	0.820	0.53	53
427	305	525	0.348	0.307	0.655	0.134	0.789	0.50	50
407	291	500	0.331	0.293	0.624	0.134	0.757	0.48	48
387	276	475	0.315	0.278	0.593	0.134	0.726	0.45	45
366	262	450	0.298	0.263	0.561	0.134	0.695	0.43	43
346	247	425	0.281	0.249	0.530	0.134	0.664	0.41	41
325	232	400	0.265	0.234	0.499	0.134	0.633	0.38	38

Again, the standard interaction formula for biaxial bending can only capture 64% of the potential capacity of the rectangular CFT column section.

CONCLUSION

The standard interaction formula for biaxial bending should no longer be included in all structural calculations since the analytical method has been proven to be the more accurate and efficient method of analysis for the capacity of rectangular steel tubular columns.

REFERENCES

Jarquio, R. V. (2004). *Analytical Method in Reinforced Concrete.* Universal Publishers, Boca Raton, Florida. 245 pp.

Jarquio, R.V. (2005). *Limitations of the Standard Interaction Formula for Biaxial Bending.* Hongkong Institute of Engineers Transactions, Volume 12, Number 2. 41-44.

Jarquio, R.V. (2005). *Yield Capacity Curve of Rectangular Steel Tubular Section.* Collaboration and Harmonization in Creative Systems. Hara, T. (editor). Taylor and Francis Group, London. (615-622).