A MATHEMATICAL MODEL FOR ESTABLISHING BUDGET LEVELS FOR A PORTFOLIO OF TRANSIT PROJECTS

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ABSTRACT

This paper proposes a mathematical model for establishing required funding levels for a portfolio of transit projects. For the past three years, the U.S. Department of Transportation has been conducting a probabilistic risk assessment on all federally-funded transit projects. For each project that requires federal funds, the risk assessment estimates the probability of cost overrun and schedule delay. The question remains that what level of confidence would be appropriate for deciding if a certain project is acceptable in terms of budget and schedule risk. At any given point in time, the Department of Transportation is funding several projects concurrently. An important question is, given a certain policy for funding an individual project (say probability of cost overrun being less than 20%), what is the probability that the total budget allocated for a portfolio of projects would be sufficient?

The paper discusses a mathematical model that is based on the project's budget, the probability of cost overrun for the project, the percent of cost overrun, and its distribution. Based on these parameters the model calculates the required budget in such a way to keep the probability of cost overrun for the portfolio of projects within acceptable levels. Inversely, the user can specify an overall probability of success (probability that the total budget for a portfolio of projects is sufficient), and the model will calculate the required probability of sufficiency of budget for the individual projects. This is an important decision because it sets the goal for the project's risk assessment.

KEY WORDS

Cost Overrun, Transit Projects, Probabilistic risk assessment, Construction Budget, Mathematical Model

INTRODUCTION

Cost and schedule overruns have plagued infrastructure capital projects sponsored by public agencies all over the world (Flyvbjerg *et al* 2003; Pickrell 1990; Dantata *et al* 2006). Several reasons have been suggested for this problem including inaccurate early cost estimates due to inexperience of estimators or the political pressure to make the project more attractive to

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sponsors, *scope creep* or additions to project that occur due to public pressure to enhance project's characteristics after the original budget has been set, and difficult underground conditions especially for transit or sewer and water project in urban areas. In response to this problem some public agencies have started to conduct probabilistic risk assessments on each major capital project to verify adequacy of budget and calculate the probability of cost overrun (Reilly *et al* 2004; Parsons 2005). The sponsor will examine the results of the risk assessment and decides on whether estimated project budget is adequate. As more risk assessments are conducted and agencies collect sufficient data, they can establish ranges or thresholds for accepting a project's budget. As an example, a sponsoring agency may decide that any project where the probability of cost overrun is larger than 25% might have inadequate budget.

Many of the sponsoring agencies are supporting several capital projects concurrently. In other words, the agency is dealing with a portfolio of projects rather than a single project and its annual budget should be allocated to these projects. A reasonable method to establish the minimum acceptable probability of cost underrun for each project is to develop a model that looks at the portfolio of projects and calculates the total budget for all projects such that the probability of overall budget shortfall remains below a certain threshold. As an example, the funding agency may be willing to accept a 10% risk that its budget may not be sufficient due to various projects' cost overruns. Based on this 90% confidence, the proposed model can suggest a probability level for individual projects. The model described in this paper helps the funding agency to decide acceptable risk levels in individual projects such that the probability of the portfolio budget shortfall will remain below a certain threshold. Sensitivity analysis on the model would allow to see the effect of various parameters on the level of funding, making this an effective decision making tool for budgeting. While the proposed model is sufficiently general, it has been developed with an interest in transit projects. Numerical example used is based on actual projects and cost figures.

THE PROPOSED MODEL

This paper discusses a mathematical model that is based on individual project's budget, the probability of cost overrun for the individual project, the percent of cost overrun, and its distribution. Based on these parameters the model calculates the required increase in budget for a portfolio of projects compared to the budgets used historically for similar projects in the past.

Project Budget *vs* Cost

We start with a model depicting the current performance characteristics of project cost *vs* budget. The assumption is that the established budget is often exceeded because of various reasons. On average, it is assumed that the cost is larger than the original budget. Using these assumptions, the model calculates the amount of shortfall for the portfolio budget.

We suggest using a hybrid shifted exponential distribution for modeling individual project cost, X_i (Fig. 1). The probability of cost underrun is α (the discrete portion of distribution), and the budget for project i is b_i. The model suggests that the probability of cost exceeding far above budget is relatively small but not insignificant. Also, the expected value of project cost is modeled as $\mu_i = \beta b_i$ where β is larger than 1.0 and can be estimated from historical data (average rate of cost overrun for a relatively large number of projects). b_i^* is the budget required to ensure that the probability of cost overrun will be limited to $1-\eta$. The general approach here to establish the cost distribution, b_i, and α based on historical data, and then quantify the needed adjustment to budget in order to get to b_i^*. The model is conservative because it does not consider the possibility of cost underrun. In the best case, the budget is equal to the cost (with probability α). The reason for this assumption is that in many cases, if the project owners sense that the project is going to be done under budget, they will try to improve and embellish the project up to the approved budget.

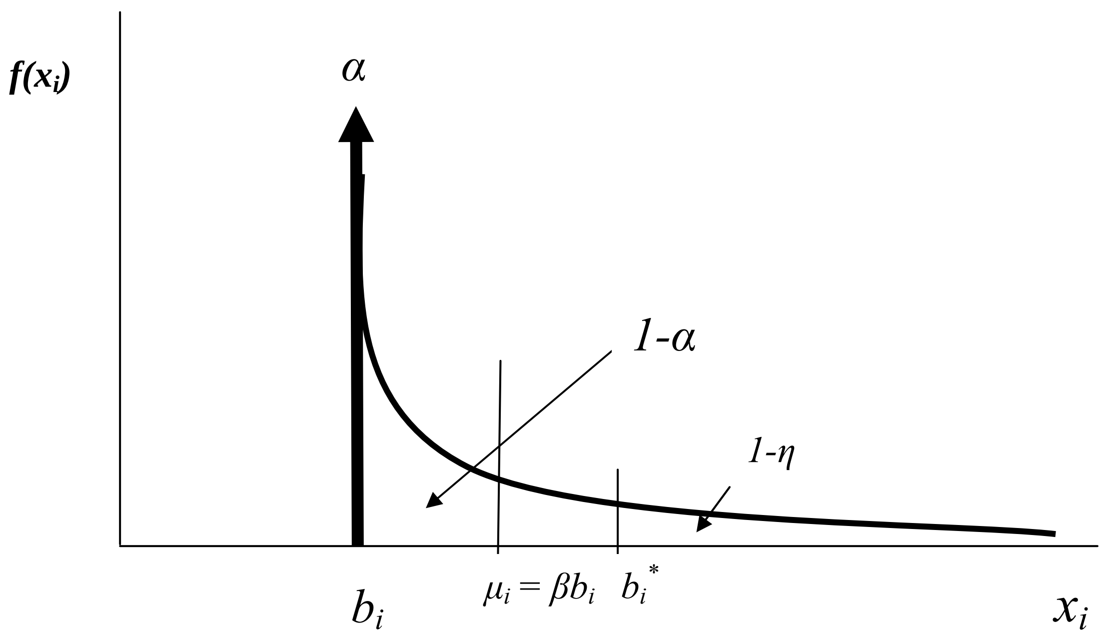

Figure 1 : The distribution of project costs, X_i

X_i is defined as follows:

$$P(X_i < b_i) = 0 \quad (1)$$

$$P(X_i = b_i) = \alpha \quad (2)$$

$$P(X_i > b_i) = 1 - \alpha \quad (3)$$

The pdf of X_i is as follows. In these equations λ_i is the parameter of the exponential distribution:

$$f(x) = \alpha \qquad for \;\; X_i \leq b_i \tag{4}$$

$$f(x) = (1-\alpha)\lambda_i e^{-\lambda_i (x-b_i)} \qquad for \;\; X_i > b_i \tag{5}$$

The mean (μ) and variance (σ^2) of X_i can be calculated from the following equations:

$$\mu_i = P(X_i = b_i)\times b_i + P(X_i > b_i)\times x_i = \alpha \times b_i + \int_{b_i}^{\infty}(1-\alpha)\lambda_i x e^{-\lambda_i (x-b_i)}dx = b_i + \frac{1-\alpha}{\lambda_i} \tag{6}$$

$$\sigma_i^2 = E(X_i^{\,2}) - \mu^2 = \alpha b_i^{\,2} + \int_{b_i}^{\infty}(1-\alpha)\lambda_i x^2 e^{-\lambda_i (x-b_i)}dx - \left[b_i + \frac{1-\alpha}{\lambda_i}\right]^2 \tag{7}$$

$$\sigma_i^2 = \frac{1-\alpha^2}{\lambda_i^{\,2}} \tag{8}$$

Now let us assume that the expected value of the final cost of a project may be modeled as a multiplier of its budget.

$$\mu_i = \beta b_i \tag{9}$$

where $\beta>1$. From there, using (6) we have,

$$\beta b_i = b_i + \frac{1-\alpha}{\lambda_i} \tag{10}$$

$$\lambda_i = \frac{1-\alpha}{(\beta-1)b_i} \tag{11}$$

Choosing a budget such as $b^*>b$ for any individual project such that the probability of cost overrun would be limited to an acceptable level $1-\eta$, we have:

$$P(X_i \leq b_i^{\,*}) = F(b_i^{\,*}) = 1-(1-\alpha)e^{-\lambda_i (b_i^{\,*}-b_i)} = \eta \tag{12}$$

Eq.(12) can be re-arranged as follows:

$$b_i^{\,*} = b_i - \tfrac{1}{\lambda_i} Ln\left[\tfrac{1-\eta}{1-\alpha}\right] = b_i + \tfrac{1}{\lambda_i} Ln\left[\tfrac{1-\alpha}{1-\eta}\right] \tag{13}$$

The original portfolio budget is $B = \sum b_i$. The new portfolio budget $B^*=\sum b_i^{\,*}$ can be calculated from (14).

$$B^* = \sum b_i^* = \sum b_i + Ln\left[\tfrac{1-\alpha}{1-\eta}\right]\sum \tfrac{1}{\lambda_i} = B + Ln\left[\tfrac{1-\alpha}{1-\eta}\right]\sum \tfrac{1}{\lambda_i} \qquad (14)$$

In the above equations, $\eta>\alpha$, so that the argument of the logarithm remains positive. This is reasonable because if $\alpha>\eta$ there would be no need to increase the total budget beyond B (the total budget is already sufficient to meet the requirement of γ probability). For the case where $\alpha = \eta$, we have $B^*=B$.

Eq. (15) can be obtained using Eq. (14) and replacing λ_i with $(1-\alpha)/[b_i(\beta-1)]$.

$$B^* = B\left\{1 + \frac{(\beta-1)}{(1-\alpha)} Ln\left[\tfrac{1-\alpha}{1-\eta}\right]\right\} \qquad (15)$$

Now consider T, the total cost of all projects. T will have an approximately normal distribution as it is the sum of several independent random variables (Central Limit Theorem). Assumption of independence is reasonable in cases where the sponsoring agency is supporting various projects in different locations, supervised/managed by different entities and built by different contractors.

$$T = \sum_i X_i \qquad \Rightarrow \qquad T \sim N(\mu_T, \sigma_T^{\ 2}) \qquad (16)$$

$$\mu_T = \sum_i \mu_i \qquad (17)$$

$$\sigma_T^{\ 2} = \sum_i \sigma_i^{\ 2} \qquad (18)$$

$$P(T \le B^*) = \Phi\left[\frac{B^* - \mu_T}{\sigma_T}\right] = \gamma \qquad (19)$$

In Eq. (19), γ is the probability of sufficiency of funds (underrun) for the portfolio of projects, and Φ is the cumulative function for standard normal distribution. Establishing an acceptable value for γ is a policy decision. Mean and variance of the total cost T can be calculated as follows.

$$\mu_T = \beta B \qquad (20)$$

$$\sigma_T^{\ 2} = \sum \sigma_i^{\ 2} = \sum \frac{1-\alpha^2}{\lambda_i^2} = \sum \frac{(1-\alpha^2)(\beta-1)^2 b_i^{\ 2}}{(1-\alpha)^2} = \frac{(1+\alpha)(\beta-1)^2}{(1-\alpha)} \sum b_i^{\ 2} \qquad (21)$$

Replacing mean and standard deviation in the Eq. (19) and dividing both sides by $(1-\beta)$ we have:

$$\frac{1}{1-\alpha} Ln\left(\frac{1-\alpha}{1-\eta}\right) = \frac{\Phi^{-1}(\gamma)}{B}\sqrt{\frac{1+\alpha}{1-\alpha}\sum b_i^{\,2}} + 1 \tag{22}$$

$$\eta = 1-(1-\alpha)e^{-\frac{\Phi^{-1}(\gamma)}{B}\sqrt{(1-\alpha^2)\sum b_i^{\,2}} - 1 + \alpha} \tag{23}$$

And from the same information, one can calculate the probability of sufficiency of the total budget γ.

$$\Phi^{-1}(\gamma) = \frac{\sqrt{1-\alpha}B}{\sqrt{(1+\alpha)\sum b_i^{\,2}}}\left[\frac{1}{1-\alpha} Ln\left(\frac{1-\alpha}{1-\eta}\right) - 1\right] \tag{24}$$

From the above equation, with knowledge of η (probability that each individual project will not overrun the budget), one can calculate the value of γ (the probability that the portfolio budget will not overrun).

APPLICATION

As an illustration of the model's application, we have studied 28 transit projects that have been funded by the U.S. Department of Transportation in the past twenty years. These projects range in cost from several hundred million to more than two billion dollars. On average the amount of cost overrun on these projects was close to 10% of original budget ($\beta = 1.1$). Also, seventeen of these projects (about 60%) exceeded their budgets ($\alpha = 0.4$). Using these values as prior data, we then reviewed the performance of a group of 30 current transit projects. For each of these projects, budget values (b_i) had been established. Using $\alpha = 0.4$ and $\beta = 1.1$ and using Eqs. (15), (23) and (24), figures (2) and (3) were prepared. Figure (2) gives the amount of B^*/B as a function of γ. As an example, if a confidence level of 80% is desired for the sufficiency of the total budget for the group of 30 projects, then a contingency of 13% is required over the base budget for the portfolio of the projects. This is in addition of any contingency that individual projects are carrying as part of their budget and is calculated based on the total portfolio budget.

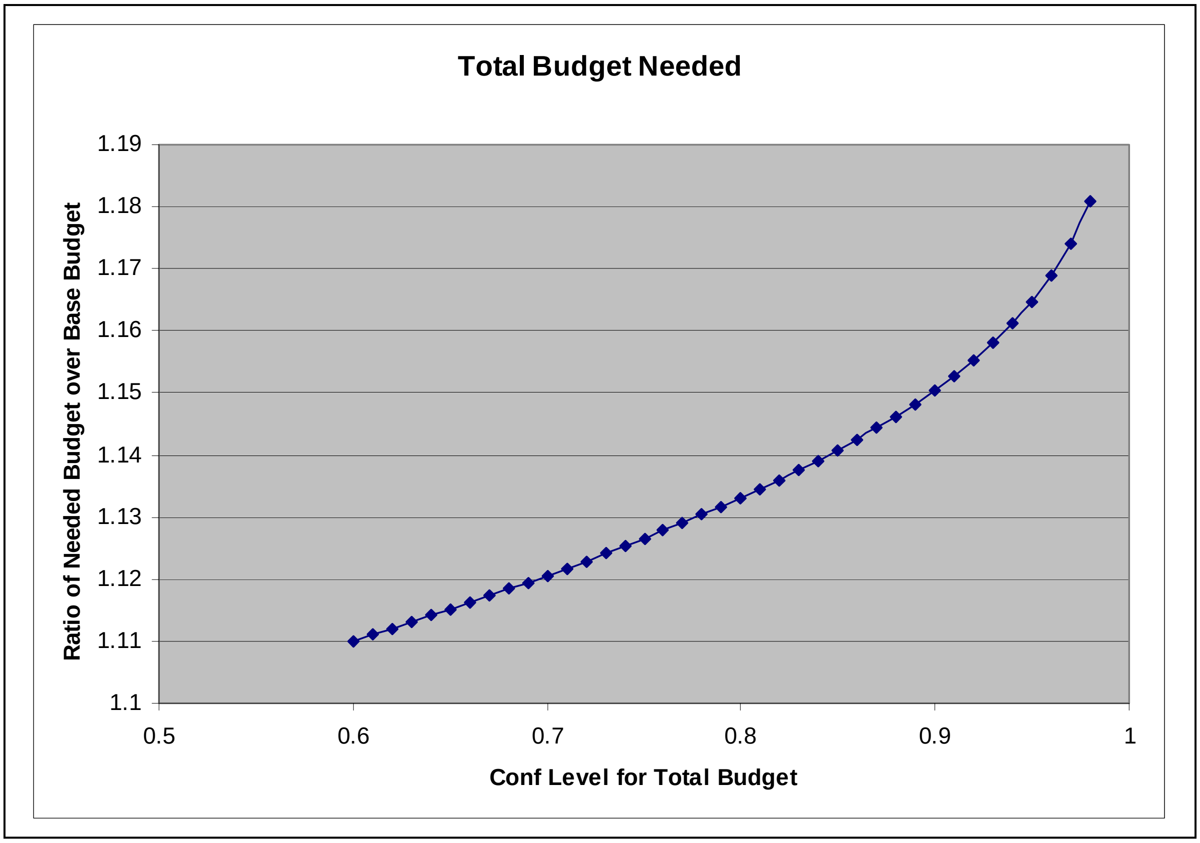

Figure 2 : Total portfolio budget needed for achieving various confidence levels

Figure (3) gives the confidence level for the portfolio of projects based on the confidence level of each individual project. As an example, if an overall confidence level of 80% is desired for the portfolio of projects (80% probability that the total budget for the 30 projects would be sufficient), then the confidence level for each individual project should be established at 73%. This means that for each probabilistic risk assessment, a target of 73% should be established for the probability of sufficiency of budget.

SUMMARY

A mathematical model has been developed that will allow the planner to see the impact of various policy decisions with respect to risk assessment of transit capital projects. An agency that is sponsoring several projects is more interested to ensure that its overall budget would be sufficient in various fiscal cycles, while an individual project may be allowed to overrun its budget. Using the portfolio approach will allow the decision maker to set realistic goals for each individual project in such a way to ensure sufficiency of funds for the portfolio of project. Furthermore, the model allows to see the effect of change in one or more variables on the final outcome in a convenient way as the modeling results closed-form solutions rather than simulation modeling.

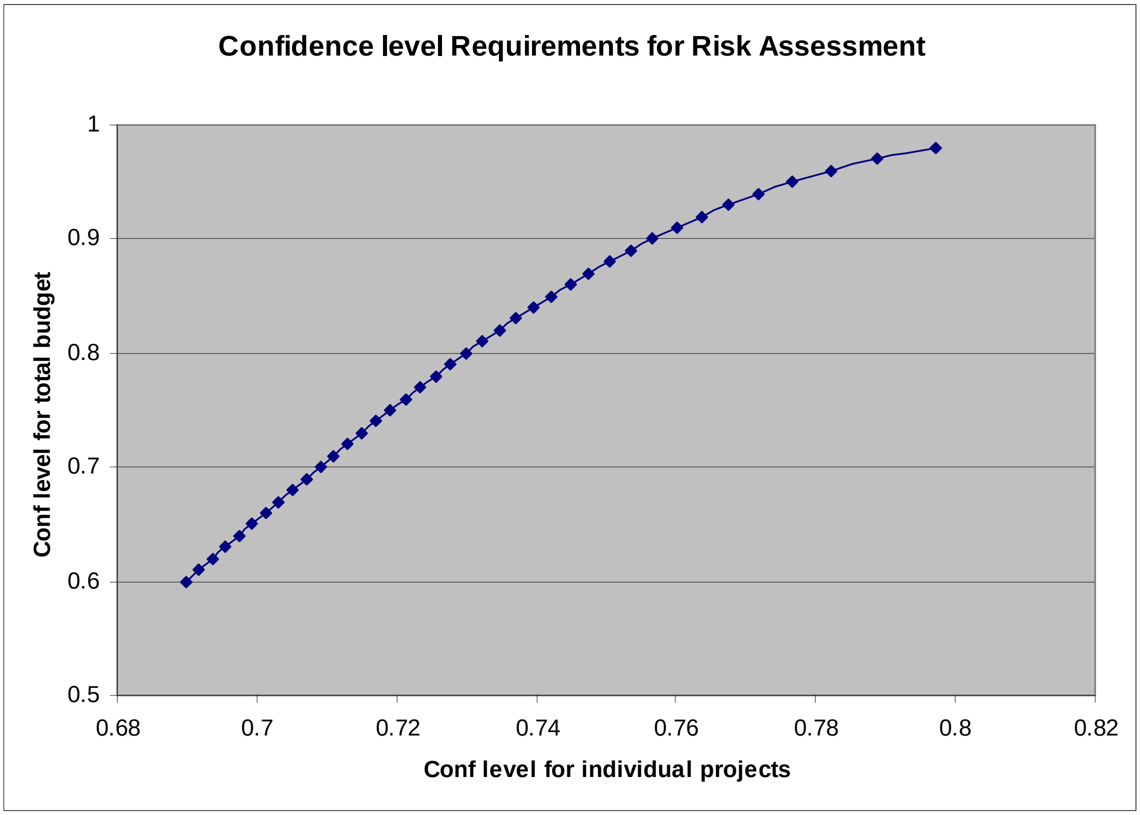

Figure 3 : The required confidence level for each individual project for achieving the desired confidence for the portfolio budget

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