INTERPRETING FUZZY COGNITIVE MAPS (FCMs) USING FUZZY MEASURES TO EVALUATE WATER QUALITY FAILURES IN DISTRIBUTION NETWORKS

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Abstract

Numerous factors affect water quality in the distribution networks and the interactions amongst them are complex and often not well understood. Water quality failures in distribution systems are scarce, which make statistically significant generalizations difficult. However, the rarity of water quality failures belies their seriousness, since each failure indicates the potential for harmful public health effects and increased public mistrust and complaints. In such data-sparse circumstances, expert knowledge and judgment can serve as an alternative source of information.

Fuzzy cognitive map (FCM) is, as the name suggests, a map of interconnected factors, each of which can interact with some or all of the others, to represent a specific process or behaviour of a network or system. A predictive model based on a FCM is a plausible way to comprehend ill-defined and complex relationships that govern water quality in the distribution network. The proposed FCM model is defined in two levels to help reduce the complexity of the system. At the modular (lower) level, rule-based FCMs are proposed for various deterioration mechanisms, which contribute to water quality failure in distribution networks. At supervisory (higher) level, a FCM is proposed, which employs fuzzy measures to interpret activation signals generated from modular FCMs to predict various types of water quality failures in distribution networks.

Key words

Fuzzy cognitive maps (FCMs), fuzzy measures, water distribution networks, water quality failure

INTRODUCTION

Complex systems that consist of a large number of interacting entities may be broken down into factors or concepts. In the paper, the terms factor and concept are used interchangeably to represent nodes of the complex system. Complex systems usually have high nonlinearity and cannot be derived from sheer summation of the behaviour of individual factors. The modelling of complex dynamic systems requires methods that combine human knowledge and experience as well as expert judgment.

Bayesian networks can represent expert knowledge in domains where knowledge is uncertain, ambiguous, and/or incomplete. Pearl (1982) claimed that classical probability theory is a reliable method to represent uncertainty, around which an expert system methodology – the Bayesian network – can be built. However, Bayesian networks have limitations in obtaining reliable conditional probabilities, and may lead to computational intractability, and inability to model vagueness and ambiguity. In Bayesian networks, all events in a system are considered equal and assigned the same binary value - yes or no. This approach has very little resemblance to most real-world problems as pointed out by Liu (2002).

Eden *et al*. (1992) defined a cognitive map as a "…directed graph characterized by a hierarchical structure which is most often in the form of a means/end graph." Cognitive maps express the judgment that certain events or actions will lead to particular outcomes. Cognitive maps have been successfully used for decision-making, prediction, explanation and strategic planning (Sadiq *et al*. 2004a).

FUZZY COGNITIVE MAPS (FCMS)

Fuzzy cognitive map (FCM), an extension of cognitive map, is an illustrative causative representation of the description and model of complex systems (Kosko 1997). FCM draws a causal representation among all identified factors or concepts of any specific system. FCM is interactive in the sense that all factors interact with each other dynamically. A complex system represented by FCM can incorporate human experience, understanding and knowledge of the system.

FCM consists of nodes (factors, concepts) and weighted arcs (causal strength, connection, edge), which are graphically illustrated as signed weighted graph(s) with optional feedback loops. Nodes on the graph represent concepts describing behavioural characteristics of the system. Concepts can be inputs, outputs, variables, states, events, actions, goals, and trends of the system. Signed weighted arcs represent causal relationships that exist among concepts.

Kosko (1997) proposed an inferencing method to calculate the value of each concept (factor) based on the influence of the interconnected concepts (factors):

$$A_j^t = f\left(k_1^i \sum_{\substack{i=1 \\ i \neq j}}^{n} A_i^{t-1} \bullet w_{ij} + k_2^j \bullet A_j^{t-1} \right) \quad 0 \le k_1^i \le 1 \quad 0 \le k_2^j \le 1 \tag{1}$$

where $w_{ij} \in$ [-1, 1] represents the causal relationship between concept i and concept j, and the negative sign represents the inverse causation. A^t_j is the normalized ($A^t_j \in [0, 1]$) value (a.k.a. activation level) of concept C_j at time step t, and $f(\cdot)$ is known as the threshold function. Generally, a sigmoid function $f(x) = \frac{1}{1+e^{-\lambda x}}$ is used to constrain the value of $f(x)$ in the interval [0, 1], where $\lambda > 0$ represents the steepness of $f(x)$. The coefficient k^i_1 expresses the influence of interconnected concepts in the configuration of the new value of concept A_i. Similarly, k^j_2 accounts for the importance of a concept itself being at its activation level in the previous time step. The selection of coefficients k^i_1 and k^j_2 depends on the nature and type of each concept, and may naturally differ from concept to concept. Kosko (1997) suggested that the previous value of each concept did not participate in the calculation of the new value of a concept (i.e., $k^j_2 = 0$) and also proposed $k^i_1 = 1$. Therefore, Equation (1) reduces to simple sigmoid transformation of weighted arithmetic mean,

$$A_j^t = f\left(\sum_{\substack{i=1 \\ i\neq j}}^{n} A_i^{t-1} \bullet w_{ij}\right) \tag{2}$$

If the FCM model has no feedback system (i.e., directed acyclic graph, DAG), the above equation can be written as

$$A_j = f\left(\sum_{\substack{i=1 \\ i\neq j}}^{n} A_i \bullet w_{ij}\right) \tag{3}$$

Alternatively, the inferencing of FCM nodes can be generalized using any disjunctive operator (or an *s*-norm) and conjunctive operator (or a *t*-norm) instead of sum-product inferencing in Equation (3). A classical example is the use of *minimum* (*t*-norm) and *maximum* (*s*-norm) operators, where Equation (3) can be re-written as

$$A_j = \max_{\substack{i=1\cdots n \\ i\neq j}}\left[\min\left(A_i, w_{ij}\right)\right] \tag{4}$$

Traditionally, causal connections in FCMs are used to describe the relationships in a forward-inferencing and monotonic way (Khan and Khor 2004). For example, if there is a positive causal link of certain strength (w_{ij}) between a causal node C_i and effect node C_j, the state value A_j will increase (decrease) with any increase (decrease) in the state value A_i. In rule-based modeling framework this concept can be translated into fuzzy rules.

For example, the negative causality (monotonically decreasing) can be represented by fuzzy rules such as: "*If* C_i is *high* (*low, medium*) *then* C_j is *low* (*high, medium*)" ($w_{ij} < 0$ in traditional FCM). Conversely, the positive causality (monotonically increasing) can be represented by fuzzy rules such as: "*If* C_i is *low* (*high*, *medium*) then C_j is *low* (*high*, *medium*)" ($w_{ij} > 0$ in traditional FCM).

However, real-world problems are often non-monotonic, which cannot always be dealt within traditional FCMs, but could be efficiently handled through fuzzy rule-based relationships (Khan and Khor 2004). For example, the non-monotonic causality can be represented by fuzzy rules such as: "*If* C_i is *low* (*high, medium*) then C_j is *low* (*low, medium or high*)".

In the case of multiple causal nodes, inferencing can be made in two possible ways, namely, rule-based FCMs i.e., either through the use of aggregation (weighting) of SISO (single-input-single-output) or MISO (multiple-inputs-single-output) fuzzy models. Proper choices are made by keeping the dimensionality issues of causal nodes in mind. The elaborations on these two models are beyond the scope of this paper (Khan and Khor 2004; Kleiner *et al*. 2005).

FUZZY MEASURES THEORY

A significant aspect of 'aggregation' is the assignation of weights to the different factors. Until recently, the most often used weighted aggregation operators were averaging operators, such as the quasi-linear means as in Equation (3). However, the weighted arithmetic means and, more generally, the quasi-linear means present some drawbacks. None of these operators are able to consider interaction between factors (concepts) in some comprehensive manner, which makes them unsuitable.

The term *fuzzy measure* was first introduced by Sugeno (1974). However, this term referred to a notion named ‘capacity’ which was first introduced by Choquet (1953). Over the years the same notion has been referred to by many different names, such as ‘confidence measure’ (Dubois and Prade 1980), ‘non-additive probability’ (Schmeidler 1986; 1989), and ‘weighting function’ (Tversky and Kahneman 1992). Complex interaction between factors (i.e., sub- and super-additive) are best introduced by assigning a non-additive set function that permits the definition of weights to a subset of factors rather than to an individual factor.

FUZZY MEASURES

It is widely accepted that *additivity* is not suitable as a required property of set functions in many real situations, due to the lack of *additivity* in many facets of human reasoning (Ross 2004). Sugeno (1974, 1977) proposed to replace the *additivity* property by a weaker one - *monotonicity* and called these non-additive (monotonic) measures ‘fuzzy measures’. However, it is important to note that fuzzy measures are non-related to fuzzy sets, typically used to express human subjectivity (Sugeno 1974).

For a discrete set x of n elements $x = \{x_1,\ldots, x_n\}$, a (discrete) fuzzy measure on θ is a set function $\mu: 2^{|x|} \to [0, 1]$ satisfying the following conditions (where $|x|$ is the cardinality of a discrete set)

- $\mu(\phi) = 0$, $\mu(x) = 1$, (where ϕ is a null subset)
- $S \subseteq T \Rightarrow \mu(S) \leq \mu(T)$. (monotonicity)

For any $S \subseteq$ X, $\mu(S)$ can be viewed as the weight or strength of the combination S for the particular decision problem under consideration. Thus, in addition to the usual weights on criteria taken separately, weights on any combination of criteria can also be defined. Monotonicity then means that adding a new element to a combination cannot decrease its importance (Marichal 1999). For example, $S =\{x_1\}$ and $T = \{x_1, x_2\}$ are the (sub)sets of $x = \{x_1, x_2, x_3\}$. The corresponding fuzzy measures, e.g., $\mu(\{x_1\}) = 0.5$ and $\mu(\{x_1, x_2\}) = 0.7$ fulfill the monotonicity condition. The fuzzy measure $\mu(\{x_1, x_2, x_3\})$ of the discrete set x (or sample space) will always be 1.

FUZZY INTEGRALS

Sugeno (1974, 1977) also introduced the concept of *fuzzy integrals* to develop tools capable of integrating all values of a function in terms of the underlying fuzzy measure (μ). An integral of fuzzy measures in a sense represents an aggregation operator, which contrary to the weighted arithmetic means, describes interactions between criteria ranging from redundancy (negative interaction, i.e., sub-additive) to synergy (positive interaction, i.e., super-additive). Several classes of fuzzy integrals exist, among which the most representatives are those suggested by Sugeno and Choquet (Marichal 1999).

The Choquet integral $C_\mu(x)$, first proposed by Schmeidler (1986) and later by Murofushi and Sugeno (1989, 1991), is based on an idea introduced in “capacity theory” by Choquet (1953). $C_\mu(x)$ is an aggregation operator, where the integrand is a set of n values $x = \{x_1,\ldots, x_n\}$. The Choquet integral of a function x with respect to μ is defined by

$$C_{\mu}(x) = \sum_{i=1}^{n} \left[x_{(i)} - x_{(i-1)} \right] \bullet \mu\left(\{ x_{(1)}, x_{(2)} \cdots x_{(i)} \right) \tag{5}$$

where $x_{(1)} \geq x_{(2)} \geq \cdots \geq x_{(n)}$ represent the order of x_i (also called utility values) in set x in descending order. The utilities $x_1, \ldots, x_n$ in our case can be replaced by activation levels of causal nodes. Therefore, Choquet integral can be used for inferencing in FCMs and can be re-written using Equations (3) and (5),

$$A_j = \sum_{i=1}^{n} \left[A_{(i)} - A_{(i-1)} \right] \bullet \mu\left(\{ A_{(1)}, A_{(2)} \cdots A_{(i)} \right) \tag{6}$$

where $\mu\left(\{ A_{(1)}, A_{(2)} \cdots A_{(i)} \right)$ are fuzzy measures similar to causal weights (w_{ij}). Interested readers should refer to Grabisch (1996) for details.

WATER QUALITY IN DISTRIBUTION NETWORKS

Water quality is generally defined by a collection of upper and lower limits on selected performance indicators (Maier 1999). A water quality failure is often defined as an exceedance of one or more water quality indicators from specific regulations, or in the absence of regulations, exceedance of guidelines or self-imposed limits driven by customer service needs (Sadiq *et al*. 2004b).

A typical modern water supply is a complex system that comprises water source(s), treatment plant(s), transmission mains, and the distribution network, which includes pipes, pumps and distribution tanks. While water quality can be compromised at any component, failure at the distribution level can be critical because it is closest to the point of delivery and, with the exception of rare filter devices at the consumer level, there are virtually no safety barriers before consumption. Water quality failures that compromise either the safety or the aesthetics of water in distribution networks, can generally be classified into the following major categories (Kleiner 1998):

- Intrusion of contaminants into the distribution network through system components whose integrity is compromised or through misuse or cross-connection or intentional introduction of harmful substances in the water distribution network
- Regrowth of microorganisms in the distribution network.
- Microbial (and/or chemical) breakthrough and by-products, and residual chemicals from the water treatment plant.
- Leaching of chemicals and corrosion byproducts from system components into the water.
- Permeation of organic compounds from the soil through system components into the water supplies.

Water distribution networks may comprise (depending on the size of the water utility) thousands of kilometres of pipes, which can vary in age, material, installation practices and operational and environmental conditions. Since the pipes are not visible, it is relatively difficult and expensive to collect data on their performance and deterioration, and therefore few field data are typically available. Further, it is often difficult to determine or validate the exact cause of a water contamination event or an outbreak of a waterborne disease because such episodes are often investigated after the occurrence has ended. This multitude of water quality failure types, combined with the inherent complexity of the distribution networks, makes risk analysis a highly challenging task and subject to substantial uncertainties.

NESTED FUZZY COGNITIVE MAP FOR PREDICTING WATER QUALITY FAILURES

We have identified more than fifty key concepts (factors) to model water quality deterioration in aging water mains. These concepts are essential for building a reliable model. Although one can conceive additional factors for FCM model, which may influence water quality in distribution networks, but it does not necessarily mean results become more reliable. We abided to the principle of Occam's razor, "one should not increase, beyond what is necessary, the number of entities required to explain anything", for the selection of concepts for the FCM model.

Figure 1 describes the complexity of proposed FCM. The model is developed using nested FCMs in two levels. At lower level, 7 modules containing MISO type rule-based FCMs for deterioration mechanisms are identified, which include potential for intrusion (PI), internal corrosion (IC), potential for leaching (PL), potential for biofilm formation (PB), disinfection loss and disinfection byproducts formation (DD), potential for permeation (PP) and water treatment breakthrough and insufficient treatment (TB). Each module contains multitude of basic concepts (nodes or factors). For example, a rule-based FCM for "potential for intrusion" is shown in Figure 1A. The arrow sign ($\rightarrow$) shows basic inputs for which the information is required. Multiple rule-bases (*Ri*) are identified in this FCM (Figure 1A) to estimate the potential for intrusion. Similar rule-based FCMs are proposed for the other deterioration mechanisms. The inferencing from each of these FCMs provides an activation signal to the supervisory FCM, which is used to predict water quality failures (Figure 1B) at the supervisory (higher) level. Discussion on the inferencing method used in rule-based FCMs is beyond the scope of this paper, however the interested readers are referred to Khan and Khor (2004).

Figure 1B describes supervisory FCM at higher level, which takes inputs from individual rule-based FCMs. Many basic concepts are common in more than one of these rule-based FCMs, which demonstrate a strong interconnectedness among the activation signals. Fuzzy measures are proposed for inferencing to account for this interconnectedness.

An aesthetic water quality failure (A-WQF) is used in this paper as an example to describe the applicability of the proposed methodology for supervisory FCM. Figure 1B shows how the activation signals from internal corrosion (IC), potential for leaching (PL), potential for biofilm formation (PB), and water treatment breakthrough (TB) feed into a node of an aesthetic water quality failure (A-WQF). Therefore, the sample space for A-WQF is θ = {IC, PL, PB, TB}. The power set $2^{|\theta|}$ requires defining 16 fuzzy measures as summarized in Table 1. Lattice representations of the power set of A-WQF are also shown in Table 1. Sub-additive (redundant) relationship refers to a case when individual rule-based FCMs contain many basic factors which are in common in the modular FCMs, whereas super-additive (synergetic) relationships refers to a case when there are very few (or none) basic factors in common. The fuzzy measures provided in Table 1 are derived here arbitrarily based on semantics (expert judgment). However, many alternative objective methods exist in the literature to derive these measures (Grabisch 1996).

Assume that the activation signals (using rule-based FCMs) for internal corrosion (IC), potential for leaching (PL), potential for biofilm formation potential (PB) and water treatment breakthrough (TB) are as 0.4, 0.1, 0.5 and 0, respectively, i.e.,

$A_{\{IC\}} = 0.4 \quad A_{\{PL\}} = 0.1 \quad A_{\{PB\}} = 0.5 \quad A_{\{TB\}} = 0$

Re-ordering is required to use Choquet fuzzy integral. The activation signals in descending orders are

$A_{(\{PB\})} = 0.5 \quad A_{(\{IC\})} = 0.4 \quad A_{(\{PL\})} = 0.1 \quad A_{(\{TB\})} = 0$

Using Equation (6), the activation level for A-WQF can be determined as follows

$$A_{\{A\text{-}WQF\}} = [A_{(\{PB\})} - A_{(\{IC\})}] \times \mu(\{PB\}) + [A_{(\{IC\})} - A_{(\{PL\})}] \times \mu(\{IC, PL\}) + [A_{(\{PL\})} - A_{(\{TB\})}] \times \mu(\{IC, PL, PB\}) + A_{(\{TB\})} \times \mu(\{IC, PL, PB, TB\})$$

$$A_{\{A\text{-}WQF\}} = [0.5 - 0.4] \times 0.3 + [0.4 - 0.1] \times 0.85 + [0.1 - 0] \times 1 + 0 \times 1 = 0.385$$

Therefore, under these conditions the A-WQF is activated at a level of ≈ 0.39.

Similarly, fuzzy measures need to be defined for physico-chemical water quality failure (PC-WQF) and microbial water quality failure (M-WQF). Note that for PC-WQF, $2^6 = 64$ fuzzy measures will be required, whereas $2^4 = 16$ fuzzy measures are required for M-WQF.

CONCLUSIONS

Numerous factors affect water quality in the distribution networks and the interactions amongst them are complex and often not well understood. Water quality failures in distribution networks are scarce, which make statistically significant generalizations difficult. A nested predictive model using fuzzy cognitive map (FCM) is proposed to comprehend these ill-defined and complex relationships that govern water quality in the distribution network. At the modular (lower) level, seven rule-based FCMs are proposed for various deterioration mechanisms, which contribute to water quality failure. At supervisory (higher) level FCM is proposed which employs fuzzy measures to interpret activation signals of modular FCMs to predict water quality failures in distribution networks.

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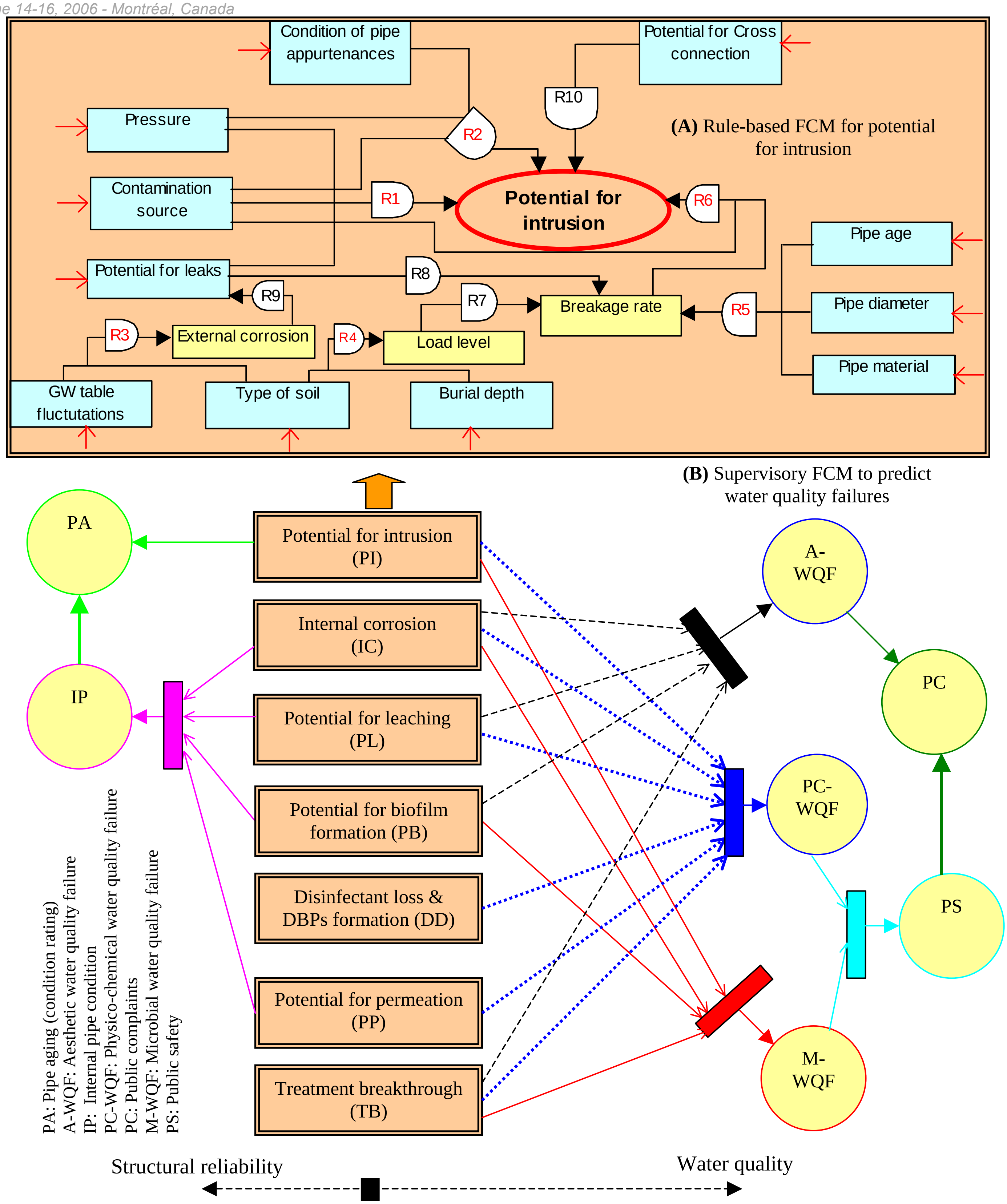
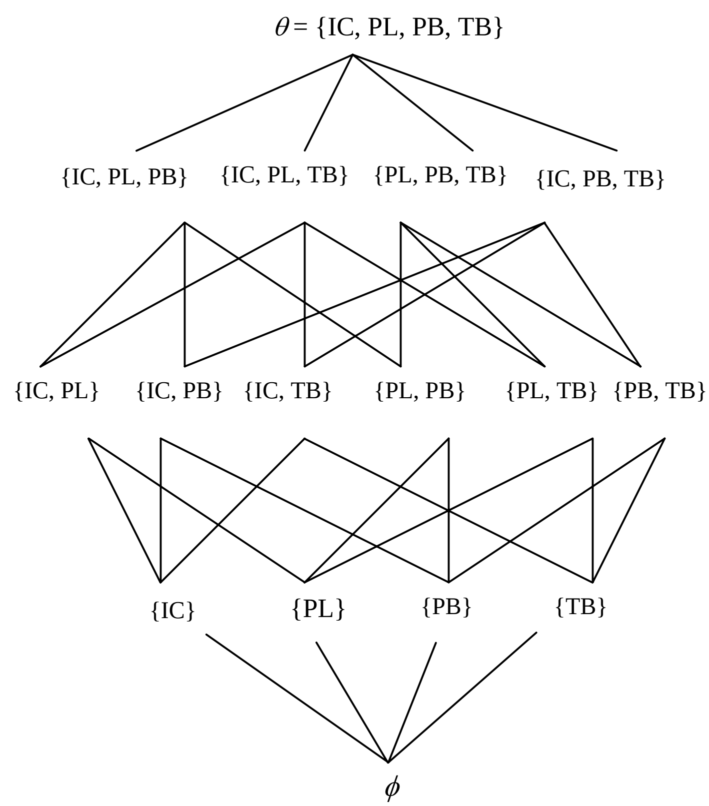


Figure 1: Proposed FCM for predicting water quality failures

Table 1: Fuzzy measures for aesthetic water quality failures (A-WQF)

Fuzzy measures	μ_i	Lattice representation of the power set of A-WQF (θ)
	0.00	
μ ({IC})	0.70	
μ ({PL})	0.40	
μ ({PB})	0.30	
μ ({TB})	0.50	
μ ({IC, PL})	0.80	
μ ({IC, PB})	0.85	
μ ({IC, TB})	0.90	
μ ({PL, PB})	0.75	
μ ({PL, TB})	0.85	
μ ({PB, TB})	0.70	
μ ({IC, PL, PB})	1.00	
μ ({IC, PL, TB})	1.00	
μ ({PL, PB, TB})	1.00	
μ ({IC, PB, TB})	1.00	
μ (θ)= μ ({IC, PL, PB, TB})	1.00	

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