WIND RESPONSE OF THE SPHERICAL STRUCTURE WITH FILM CLADDING

Edward Potetyunko[1], Angela Karsian[2], Issac Herskowitz[3], and Leonid Srubshchik[4]

ABSTRACT

In this paper we study a response of the rigid spherical structure with a film cladding subjected to stationary or time-varying wind loads caused by the thunderstorms and hurricanes. We show that the presence of the film cladding can significantly reduce the structure response to wind forces. As a result, it might improve the designer's ability to estimate wind effects on structures from the points of view both strength and serviceability.

The governing equations of the mathematical model contain coupled axisymmetric dynamic problems for partial differential equations: Navier-Stokes equations for air flow with the different angles of attack of the flow velocity and Navier-Stokes equations for the spherical layer of the film with the constant thickness. We compute wind-induced forces on structure with and without the film cladding and find values of the parameters for which the response forces take minimal values under streamlining by air flow. In this case we deduce the relation between response forces and coefficients of the kinematic viscosity for film and air, thickness of the film and radius of shell. Also we get the formula for the minimum volume of the film. Then we obtain the formula for the energy of the response forces by taking the surface integral of the scalar product of the vectors force and velocity. We develop the algorithm of the control of the film parameters providing decrease of the response forces in the structure. The solutions of the governing equations are obtained by analytical and numerical methods. The numerical results are given for specific values of the parameters.

KEY WORDS

wind loads, structure response, viscosity, film cladding, coupling, decision making

INTRODUCTION

In this paper we study a response of a rigid spherical structure with film cladding subjected to an external air flow caused by the stationary or time-varying wind loads. It is supposed that the film cladding has the constant thickness and its physical characteristics such as density

[1] Professor, Dept. Theory of Elasticity, 5 Zorge Street, Rostov State University, 344090 Russia, Phone 7-8632-922592, FAX 7-8632-284890, Potet@yandex.ru

[2] Ph.D Student, Dept. Theory of Elasticity, 5 Zorge Street, Rostov State University, 344090 Russia, Phone 7-8632-922592, FAX 7-8632-284890, Potet@yandex.ru

[3] Professor, Dept. Computer Science, 27-33 West 23rd Street, Touro College, New York, NY, 10010, Phone 1(212)463-0400 x 231, FAX 1(212)627-9144 , issacherskowitz@hotmail.com

[4] Professor, Dept. Computer Science, 27-33 West 23rd Street, Touro College, New York, NY, 10010, Phone 1(212)463-0400 x 231, FAX 1(718)265-6413, leonids@touro.edu

and viscosity are different from the corresponding characteristics of the external flow. We show that the presence of the film cladding can significantly reduce the structure response to wind forces.

FORCE EFFECT OF THE STATIONARY WIND LOAD

Consider an external axisymmetric flow of the viscous incompressible fluid over and along a stationary rigid sphere covered by a film cladding with a constant thickness h. Let ρ_1, μ_1 and $\overline{U} = const$ denote respectively the density, kinematic viscosity and velocity at infinity of the fluid external flow. The film cladding represents a spherical layer filled in by the fluid lubrication with a density ρ_2 and kinematic viscosity μ_2.
We introduce the spherical system of coordinates (r, ψ, θ) with an origin O at the center of the rigid sphere and choose the axis of symmetry of external flow at the ray $\psi = 0$. We treat a motion of the lubrication as an internal flow between external flow and a rigid sphere of the radius $r = a$ with center at the origin O. Then the motion of the both viscous fluids in the general non-stationary case is described by the system of the equations

$$\frac{\partial \overline{V}^{\pm}}{\partial t} = -\frac{1}{\rho^{\pm}} \nabla p^{\pm} + \nu^{\pm} \Delta \overline{V}^{\pm}; \ \nabla \overline{V}^{\pm} = 0; \ \overline{V}^{\pm} = (V_r^{\pm}, V_\theta^{\pm}).$$

Here $\overline{V}^{\pm}$ is a velocity vector, $p^{\pm}$ is pressure where superscripts (+) and (-) stand for description of the physical characteristics and unknown variables of the external and internal flow respectively.
Consider the boundary and interface conditions.
1) The normal and tangential components of the velocity of the lubrication at the surface of the rigid sphere $r = a$ equal zero, i.e.

$$V_r^- = 0; \ V_\theta^- = 0.$$

2) The normal and tangential components of the velocity and stresses at the "external fluid – lubrication" interface $r = a + h$ are equal, i.e.

$$V_r^- = V_r^+; \ V_\theta^- = V_\theta^+, \quad p_{rr}^- = p_{rr}^+, p_{r\theta}^- = p_{r\theta}^+$$

3) The components of the velocity of the external flow at infinity are

$$V_r \to UP_1(\cos\theta); \ \ V_\theta \to -U \frac{dP_1(\cos\theta)}{d\theta} .$$

By taking the divergence of the left-hand and right-hand sides of the motion equations and taking into account that $\nabla \overline{V} = 0$ we obtain the Laplace's equation for pressure p

$$\nabla \cdot \nabla p = \nabla^2 p = 0 .$$

In the spherical coordinates in the case of the axial symmetry this equation has a form

$$\frac{\partial}{\partial r}(r^2\frac{\partial p}{\partial r})+\frac{\cos\theta}{\sin\theta}\frac{\partial p}{\partial\theta}+\frac{\partial^2 p}{\partial\theta^2}=0$$

With $p(r,\theta)=R(r)T(\theta)$ the separation of variables and use of the Legendre polynomials $P_n(\cos\theta)$ leads to solution

$$p(r,\theta)=\sum_{n=0}^{\infty}\left(C_{1n}r^n+C_{2n}r^{-(n+1)}\right)P_n(\cos\theta)$$

We set $C_{10}=C_1$, $C_{20}=C_1$, $C_{1n}=B_n$, $C_{2n}=A_n$ and get the last expression in the form

$$p=C_0+C_1r^{-1}+\sum_{n=1}^{\infty}\left[B_nr^n+A_nr^{-(n+1)}\right]P_n(\cos\theta)$$

Then one might write the formulas for pressure in the fluid for the both flows

$$p^+=C_0^{\ +}+\frac{C_1^{\ +}}{r}+\left[B_1^{\ +}r+\frac{A^+{}_1}{r^2}\right]P_1(\cos\theta)+\sum_{n=2}^{\infty}A_n^{\ +}r^{-(n+1)}P_n(\cos\theta)\ ,\quad r\in(a+h,\infty)$$

$$p^-=C_0^{\ -}+\frac{C_1^{\ -}}{r}+\sum_{n=1}^{\infty}B_n^{\ -}r^nP_n(\cos\theta)+\sum_{n=1}^{\infty}A_n^{\ -}r^{-(n+1)}P_n(\cos\theta)\,,\qquad r\in(a,a+h)\,.$$

We substitute formula for p^+ into equation of motion of an external flow to obtain

$$\frac{\partial V^+{}_r}{\partial t}=\frac{1}{\rho}\frac{C^+{}_1}{r^2}+\frac{1}{\rho}P_1(\cos\theta)\left[-B^+{}_1+\frac{2}{r^3}A^+{}_1\right]+\frac{1}{\rho}\sum_{n=2}^{\infty}\frac{n+1}{r^{n+2}}A^+{}_nP_n(\cos\theta)+$$
$$+\nu\left[\frac{\partial^2V^+{}_r}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2V^+{}_r}{\partial\theta^2}+\frac{4}{r}\frac{\partial V^+{}_r}{\partial r}+\frac{2V^+{}_r}{r^2}+\frac{ctg\theta}{r^2}\frac{\partial V^+{}_r}{\partial\theta}\right].$$

Here $\rho=\rho^+$ and $\nu=\nu^+$. Continuity equation in spherical coordinates has a form

$$\frac{\partial V_r^+}{\partial r}+\frac{1}{r}\frac{\partial V_\theta^+}{\partial\theta}+\frac{2V_r^+}{r}+\frac{V_\theta^+ctg\theta}{r}=0\,.$$

Hence we obtain

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta^+)=-\frac{1}{r}\frac{\partial}{\partial r}(r^2V_r^+)$$

Next we find the normal and tangential components of velocity for both external and internal flows. In the stationary case we have

$$\frac{\partial V_r^{\ +}}{\partial t}=0\,,\ \frac{\partial V_\theta^{\ \pm}}{\partial t}=0\,.$$

We seek $V_r^{\ +}$ in the form $V_r^{\ +}=P_1(\cos\theta)F_r^{\ +}(r)$ and substitute $V_r^{\ +}$ into equation of motion to obtain

$$V_r^{\ +}=P_1(\cos\theta)\left(\frac{A_1^{\ +}}{\mu_1}\frac{1}{r}-\frac{D_1^{\ +}}{3r^3}+D_0^{\ +}\right),\quad r\in(a+h,\infty).$$

We note that $B_1^+ = 0$, since the velocity is restricted as $r \to \infty$.

We seek V_r^- in the form $V_r^= = P_1(\cos\theta)F_r^-(r)$ and substitute it into the corresponding equation of motion to get

$$V_r^- = P_1(\cos\theta)\left(\frac{A_1^-}{\mu_2}\frac{1}{r} - \frac{D_1^-}{3r^3} + D_0^- + \frac{B_1^-}{\mu_2}\frac{r^2}{10}\right), \quad r \in (a, a+h).$$

Now we derive the formulas for $V_\theta^\pm$. From continuity equation we have

$$\left(r\left[\frac{B_1^+}{\mu_1}\frac{2r}{10} - \frac{A_1^+}{\mu_1 r^2} + \frac{D_1^+}{r^4}\right] + 2D_0^+ + \frac{2B_1^+}{\mu_1}\frac{r^2}{10} + \frac{2A_1^+}{\mu_1 r} - \frac{2}{3}\frac{D_1^+}{r^3}\right)P_1(\cos\theta) = -\frac{\partial V_\theta}{\partial\theta} - V_\theta ctg(\theta) =$$

$$= 2F_\theta^+(r)P_1(\cos\theta)$$

We seek V_θ^+ in the form $V_\theta^+ = \frac{dP_1}{d\theta}F_\theta^+(r)$ and deduce the formulas

$$\frac{\partial V_\theta^+}{\partial\theta} + V_\theta^+ ctg\theta = \frac{d^2P_1(\cos\theta)}{d\theta^2}F_\theta^+(r) + ctg\theta\frac{dP_1(\cos\theta)}{d\theta}F_\theta^+(r) =$$

$$F_\theta^+(r)\left[\frac{d^2P_1(\cos\theta)}{d\theta^2} + ctg\theta\frac{dP_1(\cos\theta)}{d\theta}\right] = F_\theta^+(r)(-2P_1(\cos\theta))$$

$$F_\theta^+(r) = D_0^+ + \frac{1}{5}\frac{B_1^+}{\mu_1}r^2 + \frac{A_1^+}{\mu_1}\frac{1}{2r} + \frac{1}{6}\frac{D_1^+}{r^3}, \quad V_\theta^+ = \frac{dP_1(\cos\theta)}{d\theta}\left(D_0^+ + \frac{A_1^+}{\mu_1}\frac{1}{2r} + \frac{1}{6}\frac{D_1^+}{r^3}\right).$$

By analogy we get

$$V_\theta^- = \frac{dP_1(\cos\theta)}{d\theta}\left(D_0^- + \frac{A_1^-}{\mu_2}\frac{1}{2r} + \frac{1}{6}\frac{D_1^-}{r^3} + \frac{1}{5}\frac{B_1^-}{\mu_2}r^2\right).$$

Using formulas

$$p_{rr} = -p + 2\mu\frac{\partial V_r}{\partial r} \quad p_{r\theta} = \mu\left(\frac{1}{r}\frac{\partial V_r}{\partial\theta} + \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r}\right)$$

we find

$$p_{rr}^+ = -C_0^+ + P_1(\cos\theta)\left(2\mu_1\frac{D_1^+}{r^4} - \frac{3A_1^+}{r^2}\right), \qquad p_{r\theta}^+ = \mu_1\frac{dP_1(\cos\theta)}{d\theta}\frac{1}{3}\frac{D_1^+}{r^4},$$

$$p_{rr}^- = -C_0^- + P_1(\cos\theta)\left(2\mu_2\frac{D_1^-}{r^4} - \frac{3A_1^-}{r^2} - \frac{3}{5}B_1^- r\right),$$

$$p_{r\theta}^{-} = \mu_2 \frac{dP_1(\cos\theta)}{d\theta}\left(\frac{1}{3}\frac{D_1^{-}}{r^4} + \frac{3}{10}\frac{B_1^{-}r}{\mu_2}\right).$$

We use the boundary conditions to obtain the system of linear algebraic equations for unknown constants:

- $$\frac{\left(\alpha^{+} - \alpha^{-}\right)}{(a+h)} - \frac{1}{3}\frac{D_1^{-} - D_1^{+}}{(a+h)^3} - \beta^{-}\frac{(a+h)^2}{5} - 2D_0^{-} = -2U$$
- $$D_0^{-} + \frac{a^2}{10}\beta^{-} + \frac{1}{a}\alpha^{-} - \frac{1}{3}\frac{D_1^{-}}{a^3} = 0$$
- $$D_0^{-} + \frac{a^2}{5}\beta^{-} + \frac{1}{2a}\alpha^{-} + \frac{1}{6}\frac{D_1^{-}}{a^3} = 0$$
- $$\left(D_1^{+}\mu_1 - D_1^{-}\mu_2\right) = -\frac{3}{10}\mu_2 B_1^{-}(a+h)^5$$
- $$-\frac{3\left(\mu_1\alpha^{+} - \mu_2\alpha^{-}\right)}{(a+h)^2} + \frac{2}{(a+h)^4}\left(\mu_1 D_1^{+} - \mu_2 D_1^{-}\right) + \frac{3}{5}(a+h)\mu_2\beta^{-} = 0$$

$$\frac{A_1^{+}}{\mu_1} = \alpha^{+},\quad \frac{A_1^{-}}{\mu_2} = \alpha^{-},\quad \frac{B_1^{-}}{\mu_2} = \beta^{-}$$

By substituting the constants obtained into the formulas for
$V_r^{+}, V_r^{-}, V_\theta^{+}, V_\theta^{-}, p_{rr}^{-} = p_{rr}^{+},\ \ p_{r\theta}^{-} = p_{r\theta}^{+}$
we find the components of the stress tensor. Using the formula

$$W = \int_0^{\pi} \left(\cos\theta\, p_{rr} - \sin\theta\, p_{r\theta}\right) 2\pi a^2 \sin\theta\, d\theta \qquad (1)$$

we calculate the forces on the interface "film – external viscous flow" and on the surface of the rigid sphere:

$$W = 2\pi(a+h)^2\left[\frac{2}{3}\left(-\frac{3A_1^{+}}{(a+h)^2} + \frac{2\mu_1 D_1^{+}}{(a+h)^4}\right) + \frac{4}{3}\mu_1\left(-\frac{D_1^{+}}{(a+h)^4}\right)\right] = -4\pi A_1^{+},$$ where

$$A_1^+ = \mu_2 \alpha^- =$$

$$= -\frac{3U\mu_2}{2}\frac{\mu_1 a(a+h)}{\mu_1(a+h)+(\mu_2-\mu_1)a} - \frac{\beta^-}{4}\frac{\mu_2 a(a+h)\left[(a+h)^2-a^2\right]}{\mu_1(a+h)+(\mu_2-\mu_1)a} = -\frac{3U}{2}\mu_2 a\frac{1}{1+\frac{a}{a+h}\left(\frac{\mu_2}{\mu_1}-1\right)}+$$

$$+\frac{1}{4}\frac{\mu_2\left[1-\frac{1}{(a+h)^2}\right]}{\left(1+\left(\frac{\mu_2}{\mu_1}-1\right)\frac{a}{a+h}\right)}\frac{Ua(\mu_2-\mu_1)\frac{a}{a+h}\mu_1\left[\mu_1-\left(\frac{a}{a+h}\right)^2\right]}{\mu_1(\mu_1-\mu_2)\left[-\frac{2}{15}\frac{(\mu_2-\mu_1)}{\mu_1}\left(\frac{a}{a+h}\right)^6-\frac{1}{3}\frac{a^3}{(a+h)^3}+\frac{1}{30}\frac{a^5}{(a+h)^5}\right]-\frac{2}{15}\mu_1^2-V},$$

$$V=\frac{1}{5}\mu_1\mu_2+\frac{1}{5}\mu_2(\mu_2-\mu_1)\frac{a}{a+h}-\frac{3}{10}\mu_1(\mu_2-\mu_1)\frac{a}{a+h}$$

We might rewrite this formula in the form

$$W=-4\pi\left(-\frac{3U}{2}a\right)\mu_2\left[\frac{1}{1+(\gamma-1)\left[\frac{1}{1+\lambda}\right]}-\frac{1}{6}\frac{\left(1-\left(\frac{1}{1+\lambda}\right)^2\right)}{1+(\gamma-1)\left(\frac{1}{1+\lambda}\right)}\right]\frac{(1-\gamma)\left(\frac{1}{1+\lambda}\right)\left(1-\left(\frac{1}{1+\lambda}\right)^2\right)}{S},$$

$$S=(1-\gamma)\left(\frac{1}{30}\left(\frac{1}{1+\lambda}\right)^5-\frac{2}{15}(1-\gamma)\left(\frac{1}{1+\lambda}\right)^6-\frac{1}{3}\left(\frac{1}{1+\lambda}\right)^3\right)-\frac{2}{15}-\frac{1}{5}\gamma-\frac{1}{5}\gamma(\gamma-1)\left[\frac{1}{1+\lambda}\right]-$$

$$-\frac{3}{10}\left(\frac{1}{1+\lambda}\right)(\gamma-1)$$

where $\lambda=\frac{h}{a}$, $\gamma=\frac{\mu_2}{\mu_1}$. These quantities depend on the dynamic viscosity, thickness of the film, and the radius of sphere. When $\mu_1 \to \mu_2$ we get Stokes' formula $W \to W_S = 6\pi\mu_2 Ua$. Using computations we examine how W is changed for different values of the parameters λ and γ. We introduce the dimensionless response $\widetilde{W} = W/W_S$. Then we have

$$\widetilde{W} =$$

$$=\left(\frac{1}{1+\frac{\gamma-1}{1+\lambda}}-\frac{1}{6}\left(1-\frac{1}{(1+\lambda)^2}\right)^2(1-\gamma)\Bigg/\left((1+\lambda)\left(\frac{\gamma+\lambda}{1+\lambda}\right)\left((1-\gamma)\left(\frac{1}{30(1+\lambda)^5}-\frac{2\cdot(1-\gamma)}{15(1+\lambda)^6}-\frac{1}{3(1+\lambda)^3}\right)\right.\right.\right.$$

$$\left.\left.\left.-\frac{2}{15}-\frac{1}{5}\gamma-\frac{1}{5}\frac{\gamma(\gamma-1)}{1+\lambda}-\frac{3}{10}\frac{\gamma-1}{1+\lambda}\right)\right)\right)*\gamma.$$

We consider two cases: 0.1<γ<0.9 and 1.1<γ<2. The calculations and formulas show that if γ<1 the response achieves the minimum value $\widetilde{W}=0.358$ for γ=0.9, λ=0.1. Also the calculations show that for γ>1 response is increased with increasing γ and λ. The dimensionless expenditure is determined by the formula

$$q=\frac{\frac{4}{3}\pi((a+h)^3-a^3)}{\frac{4}{3}\pi a^3}.$$

For example, q=0.331 if $\gamma=0.9$ and $\lambda=0.1$.
The coefficient of the sphere response describes streamlining of sphere and its response to this streamlining. The greater response of body the more energy is spent on its motion.
Coefficient of sphere response covered by the lubricant is determined by the formula

$$C_w=$$

$$=\left(\frac{1}{1+\frac{\gamma-1}{1+\lambda}}-\frac{1}{6}\left(1-\frac{1}{(1+\lambda)^2}\right)^2(1-\gamma)/\left((1+\lambda)\left(\frac{\gamma+\lambda}{1+\lambda}\right)\left((1-\gamma)\left(\frac{1}{30}\frac{1}{(1+\lambda)^5}-\frac{2}{15}\frac{1-\gamma}{(1+\lambda)^6}-\frac{1}{3(1+\lambda)^3}\right)-\right.\right.\right.$$

$$\left.\left.\left.-\frac{2}{15}-\frac{1}{5}\gamma-\frac{1}{5}\frac{\gamma(\gamma-1)}{1+\lambda}-\frac{3}{10}\frac{\gamma-1}{1+\lambda}\right)\right)\right)*3\gamma/\mathrm{Re}$$

Coefficient of the body response is calculated by the formula $C_w=\frac{W}{F}$, where F - is a maximum area of the cross section. The response approaches 0 as $\gamma\to 0$. The response becomes extremely large as $\gamma\to\infty$
Consider the ratio of C_w to the Stokes' response coefficient C_S=3/Re:

$$\frac{C_w}{C_S}=\lambda\cdot F(\lambda,\gamma),\quad C_w=\frac{3}{\mathrm{Re}}\gamma\cdot F(\lambda,\gamma).$$

Hence we obtain 1) $\gamma<1$: if $\gamma=0.9$ and $\lambda=0.1$ then coefficient of the response is decreased by 2.6 times; 2) $\gamma>1$: if $\gamma=1.1$ and $\lambda=0.1$ then coefficient of the response is increased by1.007 times.
Now we find energy which is spent by a flow in the case of the motion of the body in the viscous fluid. By using the formula

$$E = \mu_1^{+}\left(\begin{array}{l} 2\left(\frac{\partial V_r^{+}}{\partial r}\right)^2 + 2\left(\frac{1}{r}\frac{\partial V_\theta^{+}}{\partial \theta} + \frac{V_r^{+}}{r}\right)^2 + 2\left(\frac{V_r^{+}}{r} + \frac{V_\theta^{+}}{r} ctg\theta\right)^2 + \\ + \left(\frac{1}{r}\frac{\partial V_r^{+}}{\partial \theta} + \frac{\partial V_\theta^{+}}{\partial r} - \frac{V_\theta^{+}}{r}\right)^2 \end{array} \right) \qquad (2)$$

we deduce the formula for an energy of flow streamlining sphere covered by film of the thickness of h:

$$E = 3\gamma^3 *$$

$$\frac{\left(\frac{1}{1+\frac{\gamma-1}{1+\lambda}} + \frac{\left(1-\frac{1}{(1+\lambda)^2}\right)^2 (1-\gamma)}{\left(6\left(\frac{\gamma+\lambda}{1+\lambda}\right)(1+\lambda)\left((1-\gamma)\left(\frac{1}{30(1+\lambda)^5} - \frac{2(1-\gamma)}{15(1+\lambda)^6} - \frac{1}{3(1+\lambda)^3}\right) - \frac{2}{15} - \frac{\gamma}{5} - \frac{\gamma}{5}\frac{(\gamma-1)}{(1+\lambda)} - \frac{3}{10}\frac{(\gamma-1)}{(1+\lambda)}\right)\right)} \right)^2}{(1+\lambda)^4}$$

$$+\frac{4}{(1+\lambda)^8} - 6\left(\frac{1}{1+\frac{\gamma-1}{1+\lambda}} + \frac{\left(1-\frac{1}{(1+\lambda)^2}\right)^2 (1-\gamma)}{(S)} \right) * \frac{\gamma}{(1+\lambda)^6}$$

where

$$S = 6\left(1+\frac{\gamma-1}{1+\lambda}\right)(1+\lambda)$$

$$*\left((1-\gamma)\left(\frac{1}{30(1+\lambda)^5} - \frac{2(1-\gamma)}{15(1+\lambda)^6} - \frac{1}{3(1+\lambda)^3}\right) - \frac{2}{15} - \frac{\gamma}{5} - \frac{\gamma}{5}\frac{(\gamma-1)}{(1+\lambda)} - \frac{3}{10}\frac{(\gamma-1)}{(1+\lambda)}\right)$$

If the cladding film is absent the formula for energy spent by the external flow has a form

$$E = U^2\pi\mu\left(-60a^4r^4 + 28a^6r^2 + 4a^3r^4 - 18a^2r^5 - 6a^4r^3 + 4a^6r + 9a^2r^4 + 12a^4r^2 - 12ar^5 - \right.$$
$$\left. -8a^3r^3 + 4r^8 - 8r^7 + 4a^6 + 4r^6 + 4r^5a^3 + 24r^6a - 12r^7a + 36a^2r^6\right)/8r^{10}$$

In the case of the sphere of the radius $r = a$ an energy of the streamlined flow is $E = \frac{9U^2\pi\mu_1}{8a^2}$.

FORCE EFFECT OF THE NONSTATIONARY WIND LOAD

Consider an external axisymmetric flow of the viscous incompressible fluid over and along a stationary rigid sphere covered by a film cladding with a constant thickness h. Let ρ_1, μ_1 and $\overline{U} = \overline{U}(t)$ denote respectively the density, kinematic viscosity and given velocity at infinity of the fluid external flow. The film cladding represents a spherical layer filled in by the fluid lubrication with a density ρ_2 and kinematic viscosity μ_2. Then the motion of the both viscous fluids in the general non-stationary case is described by the system of the equations

$$\frac{\partial \overline{V}^{\pm}}{\partial t} = -\frac{1}{\rho^{\pm}}\nabla P^{\pm} + \nu^{\pm}\Delta \overline{V}^{\pm};\ \operatorname{div}\overline{V}^{\pm} = 0;\ \overline{V}^{\pm} = (V_r^{\pm}, V_\theta^{\pm}).$$

Here $\overline{V}^{\pm}$ is a velocity vector, $p^{\pm}$ is pressure where superscripts (+) and (-) stand for description of the physical characteristics and unknown variables of the external and internal flow respectively.

Consider the boundary and interface conditions.

1) The normal and tangential components of the velocity of the lubrication at the surface of the rigid sphere $r = a$ equal zero, i.e.

$$V_r^{-} = 0;\ V_\theta^{-} = 0.$$

2) The normal and tangential components of the velocity and stresses at the "external fluid – lubrication" interface $r = a + h$ are equal, i.e.

$$V_r^{-} = V_r^{+};\ V_\theta^{-} = V_\theta^{+},\quad p_{rr}^{-} = p_{rr}^{+}, p_{r\theta}^{-} = p_{r\theta}^{+}$$

3) The velocities of the external flow at infinity are

$$V_r \to U(t)P_1(\cos\theta);\quad V_\theta \to -U(t)\frac{dP_1(\cos\theta)}{d\theta}\ .$$

By taking the divergence of the left-hand and right-hand sides of the motion equations and taking into account that $\nabla\overline{V} = 0$ we obtain the Laplace's equation for pressure p

$$\nabla\cdot\nabla p = \nabla^2 p = 0\,.$$

Using the Laplace transform to equations of motion we deduce the formulas for pressure in an internal region $r \in (a, a+h)$ and an external region $r \in (a+h, \infty)$ respectively

$$\hat{p}^{-} = \left(\hat{C}_0^{-} + \frac{\hat{C}_1^{-}}{r}\right)P_0(\cos\theta) + \left(\hat{D}_1^{-} r + \frac{\hat{A}_1^{-}}{r^2}\right)P_1(\cos\theta) + \sum_{n=2}^{\infty}\left(\hat{D}_n^{-} r^n + \frac{\hat{A}_n^{-}}{r^n}\right)P_n(\cos\theta)$$

$$\hat{p}^{+} = \left(\hat{C}_0^{+} + \frac{\hat{C}_1^{+}}{r}\right)P_0(\cos\theta) + \left(\hat{D}_1^{+} r + \frac{\hat{A}_1^{+}}{r^2}\right)P_1(\cos\theta) + \sum_{n=2}^{\infty}\left(\frac{\hat{A}_n^{+}}{r^n}\right)P_n(\cos\theta)\ .$$

Now we find the Laplace Transform of the normal and tangential components of the velocity for the external and internal flows.

We use the boundary conditions to obtain and solve the system of linear algebraic equations for unknown constants. We substitute these constants into the formulas for $p_{rr}^{-}, p_{rr}^{+},\ \ p_{r\theta}^{-}, p_{r\theta}^{+}$, take the Laplace transform for each component of the stress tensor and solve the system of the linear equations obtained. Then we apply inverse Laplace transform to get the normal and tangential components of the velocity for the external flow.

$$V_r(r)=\int_0^{\tau} U(\tau)\cos(\theta)*$$

$$*\left(Dirac(\tau)+\frac{\left(-a^2 Dirac(\tau)+3\nu\left(-1+\sqrt{\frac{\nu}{4\iota\tau}}\exp^{\left(-\frac{(r-a)^2}{4\nu\tau}\right)} r+erfc\left(\frac{r-a}{2\sqrt{\nu\tau}}\right)-a\sqrt{\frac{\nu}{\pi\tau}}\right)\right)a}{r^3}\right)d\tau$$

$$V_\theta(\theta)=\int_0^{\tau}\frac{1}{2}U(\tau)\sin(\theta)*$$

$$*\left(-2Dirac(\tau)+\right.$$

$$\left.+\frac{a\left(\frac{3}{2}\frac{(r-a)\exp^{\left(-\frac{(r-a)^2}{4\nu\tau}\right)} r^2}{\sqrt{\pi}\sqrt{\nu}\tau^{\frac{3}{2}}}-a^2 Dirac(\tau)-3a\sqrt{\frac{\nu}{\pi\tau}}+3\nu\left(erfc\left(\frac{r-a}{2\sqrt{\tau\nu}}\right)-1\right)+3\sqrt{\frac{\nu}{\tau\pi}}\exp^{\left(-\frac{(r-a)^2}{4\nu\tau}\right)} r\right)}{r^3}\right)d\tau$$

Using the formula (1) we evaluate the force response on the interface “film – viscous external flow”. Using the formula (2) we find the energy spent by the nonstationary flow in the case of streamlining of the body covered by the film. This expression provides the formula for energy spent by the nonstationary fluid flow in the case of streamlining of the sphere without film cladding as the thickness of the film approaches to zero and kinematic viscosity of an external fluid approaches to the kinematic viscosity of an internal fluid.

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