PREDICTING THE PUNCHING SHEAR STRENGTH OF INTERIOR SLAB-COLUMN CONNECTIONS USING FUZZY SYSTEMS

Mahmoud Reda Taha[1], Kyoung Kyu Choi[2], and Alaa Sherif[3]

ABSTRACT

This research presents an alternative approach for predicting the punching shear strength of interior slab-column connections using fuzzy logic (FL). A total of 176 data points were used in the training and testing of the fuzzy system. The data was obtained from test results of concentric punching shear tests of reinforced concrete flat plates available in the literature. The fuzzy system was trained to address the uncertainty in the relationship between various parameters, which might not be captured in previous research attempts. The fuzzy-based model was verified by using the remaining data sets that were not used in the training process. The model predictions were compared to current strength models most widely used in design practice such as CEB-FIP MC 90, Eurocode 2, and ACI 318 codes. It was found that a significant enhancement in the prediction of the punching shear strength of interior slab-column connections can be achieved by means of the fuzzy system.

KEY WORDS

Punching Shear, Slab-Column Connections, Fuzzy Systems, Uncertainty.

INTRODUCTION

Flat plates are widely used worldwide for their economic and fuctional advantages such as fast construction, low story height, and irregular column layout. Geometrically, the system consists simply of slabs directly supported on the columns. In spite of their simple appearance, from a structural point of view, flat plates are complex structures. Furthermore, a flat plate usually fails in a brittle manner by punching at the slab-column connections within the discontinuity region known as D-region (Schlaich et al. 1987). At the connection, three dimensional stresses are developed due to combined high shear and normal stresses, which are very complicated to analyze accurately (CEB-FIP 2001).

For the last four decades a lot of research has been performed in order to solve this complex problem of concentric punching shear of reinforced concrete flat plates. The results range from mechanical models up to purely empirical models. Kinnunen and Nylander

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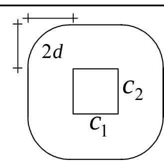
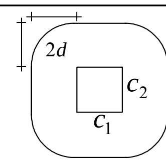
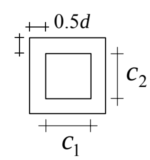
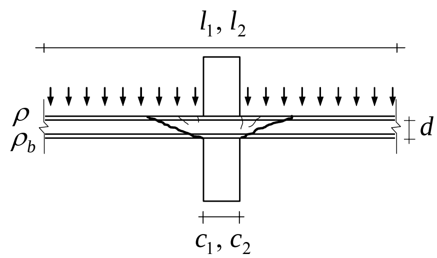
(1960), and Bažant and Cao (1987) developed design equations based on failure criteria and fracture mechanics, respectively. Pralong (1982) and Nielsen (1999) developed strength models based on the theory of plasticity. Alexander and Simmonds (1987) proposed a strut-and-tie model with concrete ties to describe the load transfer in the slab-column connections. However, these theoretical strength models are too complicated to use in design practice. Furthermore, such complexity can be hardly justified because of the low accuracy of punch shear strength prediction (Theodorakopoulos and Swamy 2002).

Design methods	CEB-FIP MC 90 (1993)	Eurocode 2 (2002)	ACI 318 (2005)
Punching shear strength (MPa)	$v_c = 0.18kf'_c{}^{1/3}\sqrt{100\rho}$ $k = 1+\sqrt{200/d} \le 2.0$ d in mm. f'_c in MPa.	$v_c = 0.18k(f'_c 100\rho)^{1/3}$ $k = 1+\sqrt{200/d} \le 2.0$ d in mm. f'_c in MPa. $\rho \le 0.02$	Minimum of $v_c = 0.33\sqrt{f'_c}$ $v_c = (0.167+\frac{0.33}{\beta_c})\sqrt{f'_c}$ $v_c = (\frac{3.32d}{b_o}+0.167)\sqrt{f'_c}$ f'_c in MPa.
Critical section, b_o			



l_1, l_2 = Span lengths of a slab
c_1, c_2 = Sizes of a rectangular column
β_c = Ratio of long side to short side of a column-section
ρ = Tension reinforcement ratio
ρ_b = Compression reinforcement ratio
d = Effective depth of a slab
b_o = Perimeter of the critical section of a slab-column connection

Figure 1: Current design codes for punching shear

To develop simple strength equations, most design codes use the so-called control perimeter approach depicted in Figure 1 based on the calibration of existing test results. The applied punching shear stress is calculated at a defined critical perimeter and compared to an allowed value. The various design codes significantly differ in defining the location of the critical section and the punching shear resistance. The punching shear strength specified in CEB-FIP MC 90 (1993) and Eurocode 2 (2002) are smaller than that specified in ACI 318 (2005), while the critical sections specified in CEB-FIP MC 90 and Eurocode 2 are much greater than that specified in ACI 318. Figure 2 shows the punching shear strength predicted by these current design methods compared with test results. In this figure, except for Eurocode 2, the current design methods show considerably large scatter represented by high standard deviations of test-prediction ratios.

It becomes obvious that the complexity of the punching problem and the dependence of the punching shear strength on a number of interacting variables necessitate the use of empirical coefficients/equations in modeling punching shear strength. A robust model for predicting punching shear strength that considers uncertainties in the modeling variables is needed.

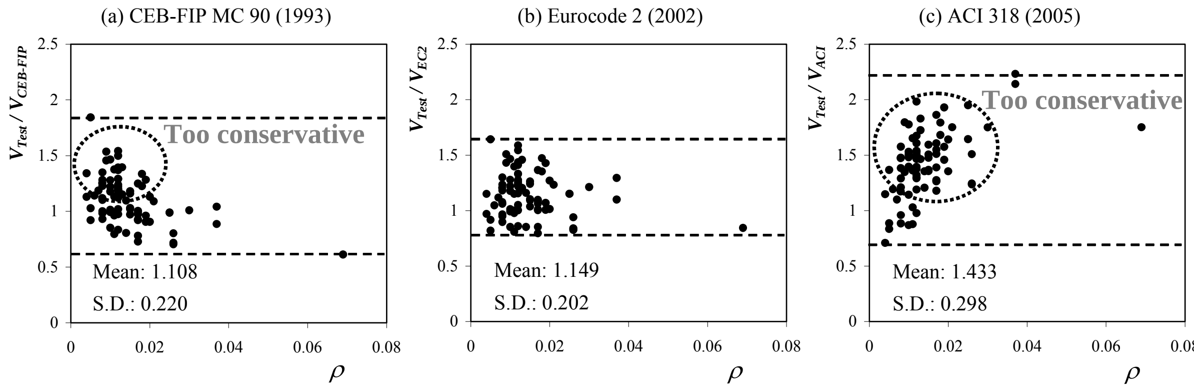


Figure 2: Punching shear strength test result to design codes prediction ratios.

The current study introduces a new approach for predicting the punching shear strength of concentrically loaded interior slab-column connections using fuzzy systems. The fuzzy-based model targets predicting the shear strength of the slab-column connections based on concrete strength, tension reinforcement ratio, and slab and column geometry. The fuzzy system was trained using an extensive punching shear test database to address the uncertainty in the relationship between the different parameters which was not usually captured in previous research attempts. The fuzzy model is then tested using parts of the database that were not used for its training process. Punching shear strength predictions of the fuzzy-based model will be compared to CEB-FIP MC 90, Eurocode 2, and American ACI 318 codes predictions.

FUZZY SYSTEMS FOR PUNCHING SHEAR STRENGTH

Fuzzy systems have been successfully used in the last decade for modeling complex engineering systems (e.g. Reda Taha et al. 2003 and Chatterjee et al. 2005) and proven as universal approximators (Kosko 1993). The capability of fuzzy systems to model complex systems is attributed to their inherent ability to incorporate uncertainty due to vagueness and/or ambiguity in modeling parameters (Klir et al 1995). The fundamental concept in complex system modeling using fuzzy systems is to establish a fuzzy rule-base that is capable of describing the relationship between the input and the output parameters (Ross 2004). This fuzzy rule-base captures individual and group relationships which distinguish the internal complex relations in the system (Passino et al. 1998). As such, system non-linearity is not recognized by using a specific power of a non-linear equation but through establishing a number of fuzzy rules such that the fuzzy system becomes capable of describing the system to a pre-specified level of accuracy (Ross 2004). A group of successful techniques to establish fuzzy rule-base has been reported in the literature (Passino et al. 1998 and Jang et al. 1997).

We start by defining N number of fuzzy sets $\underset{\sim}{A}$ over the domain of the input parameter (x). Each value of the parameter (x) has a membership to each fuzzy set $\underset{\sim}{A}$. The concept of membership (degree of belonging) represents a cornerstone in the formulation of fuzzy set theory. The membership is denoted $\mu_{\underset{\sim}{A}}(x)$ and ranges between 0.0 and 1.0. Membership values do not express probability but characterize the evaluator's view of the extent to which a parameter belongs to the fuzzy set (Laviolette et al. 1995). Several methods for establishing membership functions are provided in the literature (Cox 1994, Ross 2004).

Here, we demonstrate the use of fuzzy set theory to model the punching shear strength of slab-column connections. Extensive preliminary investigations were performed to identify the most important input parameters that have a significant influence on the punching shear strength. Possible parameters included concrete compressive strength, slab thickness and effective depth, span length, column geometry, punching shear perimeter, and compression and tension reinforcement ratio. The investigation showed that for circular and rectangular columns with (c_1/c_2) ratio equals to 1.0 and perimeter to slab depth ratio (b_0/d) ranging between 5.8 and 20.8, the most significant parameters that affect the punching shear strength are: concrete compressive strength (f'_c), slab thickness (h) and tension reinforcement ratio (ρ). These three parameters have therefore been selected to model punching shear strength in the proposed fuzzy-based model.

In the present study, the punching shear failure load of slab-column connection without shear reinforcement (V_c) has been defined as

$$V_c = v_c \;\; b_0 \; d \tag{1}$$

V_c = punching failure load; b_o = critical perimeter at a distance $d/2$ from the column face = ($2c_1 + 2c_2 + 4d$) for a rectangular column, $\pi(D+d)$ for a circular column; c_1, c_2 = column sizes of a rectangular column; and D = diameter of a circular column. Equation (1), although simplified, respects the fundamental punching failure mechanism observed by most researchers. The modeling process starts by fuzzifying all three input domains and constructing a fuzzy rule-base that describes the relationship between the fuzzy sets defined on all input domains and the punching shear strength. Examplar rule in the fuzzy rule-base can be defined as

$$\text{If } f_c \in \underset{\sim}{A}_f^k,\; h \in \underset{\sim}{A}_f^k \text{ and } \rho \in \underset{\sim}{A}_\rho^k \text{ Then } v_i = a_i f_c + b_i h + c_i \rho + d_i \tag{2}$$

where $\underset{\sim}{A}_f^k, \underset{\sim}{A}_h^k$ *and* $\underset{\sim}{A}_\rho^k$ are the kth fuzzy set (k = 1, 2 or N_j) defined on the universe of discourses of compressive strength f'_c, slab height h and reinforcement ratio ρ respectively. N_j is the total number of fuzzy sets defined over the jth input parameter. Equation (2) represents the ith rule in the fuzzy rule-base. a_i, b_i, c_i and d_i are known as the consequent parameters that define the output side of the ith fuzzy rule. If the fuzzy rule-base includes R rules, the punching shear strength v_c can be computed as

$$v_c = \left(\sum_{i=1}^{R} \lambda_i v_i \right) / \left(\sum_{i=1}^{R} \lambda_i \right) \tag{3}$$

where v_i is the output of the i[th] rule in the fuzzy rule-base and λ_i represents the weight of the i[th] rule in the fuzzy rule-base. If we employ bell-shape membership functions (MF) (Jang et al. 1997) to describe the three input parameters, the weight of the i[th] rule in the fuzzy rule-base can be computed as

$$\lambda_i = \frac{\prod_{j=1}^{T} \dfrac{1}{1 + \left| \dfrac{x_j - x_{c\,j}^{\,k}}{w_j^k} \right|^{2q_j^k}}}{\sum_{i=1}^{R} \prod_{j=1}^{T} \dfrac{1}{1 + \left| \dfrac{x_j - x_{c\,j}^{\,k}}{w_j^k} \right|^{2q_j^k}}} \quad \text{for } i = 1 \ldots R \tag{4}$$

where $x_{c\,j}^{k}$ w_j^k and q_j^k represents the center, the top width and the shape parameter of the membership function defining the k[th] fuzzy set defined over the j[th] input parameter. The fuzzy operator (Π) represents the T-norm (minimum-multiplication) operation (Gupta et al. 1991) to capture the interaction between the input parameters and its influence on the output. T represents the total number of input parameters (here T = 3). The number of fuzzy rules "R" is a function of the number of input variables (T) and the number of fuzzy sets (N_j) defined over each input domain.

The learning process starts by initializing the premise parameters (parameters describing the membership functions $x_{c\,j}^{k}$ w_j^k and q_j^k) using a fuzzy clustering algorithm (Bezdek 1981). This is followed by computing the consequence parameters (a_i, b_i, c_i and d_i) using least square techniques (Partington et al. 1995) such that the root mean square error of the punching shear strength does not exceed a target root mean square error ($RMSE_T$) (here $RMSE_T$ = 1E-5). To further enhance the learning process, the premise parameters are then updated using a back propagation technique and the updated premise parameter are used to re-compute a new set of consequence parameters. The process continues and the fuzzy rule-base parameters (premise and consequent parameters) are updated in each training epoch until the target root mean square error or a maximum number of training epochs is reached.

For training and testing of the fuzzy-based model, one hundred seventy six test specimens performed by eighteen researchers as reported in the FIP bulletin 12 (2001) were used. We only considered specimens that were reported to fail in punching shear only. Specimens had two types of boundary geometries (circular and rectangular flat plates) and two types of column shapes (circular column, and rectangular columns with $c_1 / c_2 = 1.0$). The dimensions and properties of the specimens are summarized in Table 1. The test specimens had a broad range of design parameters: $8.2 \le f'_c \le 119.0$ (MPa), $80 \le h \le 320$ (mm), $0.3 \le \rho \le 8.5$ (percent), and $5.8 \le b_o / d \le 20.8$. b_o = critical perimeter at a distance d/2 from the column face. These data cover a wide range of the material and geometric properties of flat plates. A specimen reported by Lovrovich and McLean (1990)'s was excluded in this study because its span length was extremely short $(l_1 / c_1 = 2)$. Another specimen by Yitzchaki (1966)'s was also excluded because its test result was significantly differed from other specimens with similar geometry and properties. 96 specimens were used for training of the fuzzy-based model. On the other hand, 80 specimens were used for testig of the model. 176 specimens, in total, were used for developing and verification of the fuzzy-based model (Table 1). All specimens used in the testing were not used in the developing the fuzzy based-model.

Table 1: Dimensions and properties of specimens, and strength-predictions

Investigator[1]	Number of specimens		f'_c (MPa)	h (mm)	ρ (percent)	$\left(\frac{v_{c,Test}}{v_{c,pred}}\right)^{(2)}$
	Training	Verification				
Hallgren and Kinnunen (1993a), Hallgren and Kinuunen (1993b), Hallgren (1996)	3	3	79.5-108.8	239-245	0.3-1.2	1.00-1.08
Tomaszewicz (1993)	9	4	64.3-119.0	120-320	1.5-2.6	0.93-1.17
Ramdane (1996), Regan et al. (1993)	7	1	23.7-89.4	125	0.6-1.3	1.28
Marzouk and Hussein (1991)	8	8	30.0-80.0	90-150	0.4-2.1	1.06-1.33
Lovrovich and McLean (1990)	2	2	39.3	100	1.7	0.89-0.95
Tolf (1988)	4	4	20.1-25.1	120-240	0.4-0.8	0.90-1.23
Regan (1986)	11	11	8.4-37.5	80-250	0.8-2.4	0.65-1.28
Swamy and Ali (1982)	1	1	37.4-40.1	125	0.6-0.7	0.99
Marti et al. (1977), Pralong et al. (1979)	1	1	23.1-30.4	180-191	1.2-1.5	0.90
Schaefers (1984)	1	1	23.1-23.3	143-200	0.6-0.8	1.00
Ladner et al. (1977), Schaeidt et al. (1970), Ladner (1973)	3	3	26.4-29.5	110-280	1.2-1.8	0.90-1.29
Corley and Hawkins (1968)	1	1	44.4	146	1.0-1.5	0.69
Bernaert and Puech (1996)	9	9	14.0-41.4	140	1.0-1.9	0.67-1.33
Manterola (1966)	6	3	24.2-39.7	125	0.5-1.4	0.66-0.96
Yitzhaki (1966)	6	6	8.2-19.0	102	0.5-8.5	0.88-1.26
Moe (1961)	7	7	20.5-35.2	152	1.1-2.6	0.66-1.13
Kinnunen and Nylander (1960)	6	6	21.6-27.7	149-158	0.5-2.1	0.72-1.29
Elstner and Hognestad (1956)	11	9	9.2-35.6	152	0.5-6.9	0.83-1.13
Total	96	80	8.2-119.0	80-320	0.3-8.5	**Mean: 1.018** **S.D. : 0.172**

(1) The properties and dimensions of these test specimens were collected from FIP bulletin 12 (2001).
(2) $v_{c,Test}$ and $v_{c,pred}$ = test results and strengths predicted by fuzzy-based model, respectively.

All modeling parameters were normalized to their maximum values determined from all database (176 data sets). The normalization process is necessary to avoid the influence of numerical weights on the learning process (Berenji et al.1992). The fuzzy rule-base that achieved the lowest root mean square error during training was used for testing and verification of the model capability to predict punching shear strength in slab-column connections. Two membership functions were used to represent each input parameters ($N_1 = N_2 = N_3 = 2$). The initial and final membership functions are shown in Figure 3 for the concrete compressive strength for exemplar demonstration of the update process during learning. A total number of eight rules (R = 8) were needed to describe the relationship between the compressive strength of concrete, the slab thickness, the tension reinforcement ratio, and the punching shear strength.

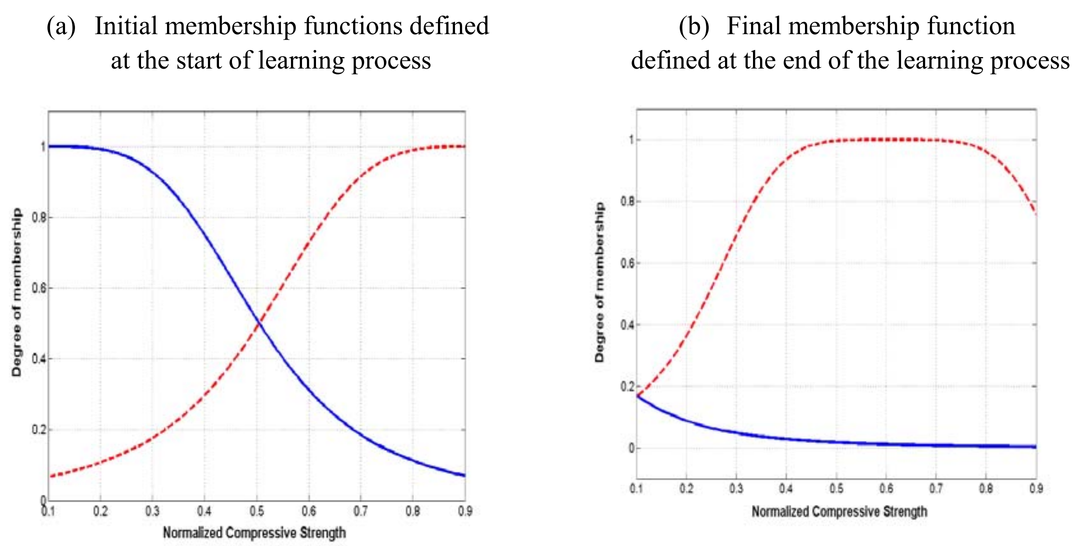


Figure 3: Membership functions used to describe the concrete compressive strength at the start (before training) and at the end (after training)

RESULTS AND DISCUSSION

It is important to emphasize that the fuzzy-based model was trained using punching shear strength testing database with specific geometrical limits: circular and rectangular columns with (c_1/c_2) ratio equals to 1.0, and slabs with perimeter to slab depth ratio (b_0/d) ranging between 5.8 and 20.8. Therefore, the results presented here are only valid for these given geometrical limits. The fuzzy-based model can be updated (re-trained) for its application to wider geometrical range beyond those mentioned here once testing database becomes available. Table 1 presents a summary of punching shear strengths of the specimens predicted by the fuzzy-based model. Figures 4 shows the ratio between the punching shear strengths test results to punching shear strengths as predicted by the fuzzy-based model. The mean value of strength test to prediction ratios is 1.018 and the standard deviation of the ratios is 17.2 percent. As shown in Fig. 4, the fuzzy-based model accurately predicted the punching shear strengths of the test specimens with a wide range of values of the design parameters. The results show that the fuzzy system can be used to predict the punching shear strengths of slab column connections with various slab geometries including thickness, and with various column shapes circular or rectangular. Moreover, prediction accuracy of the proposed fuzzy-based model is higher than that of the CEB-FIP MC 90 (1993), the Eurocode 2 (2002) and ACI 318-02 (2005), whose mean values of strength ratio ($V_{test} / V_{predicted}$) were found to be 1.108, 1.149 and 1.433, respectively with standard deviations of 22 percent, 20.2 percent and 29. 8 percent respectively as shown in Figure 2.

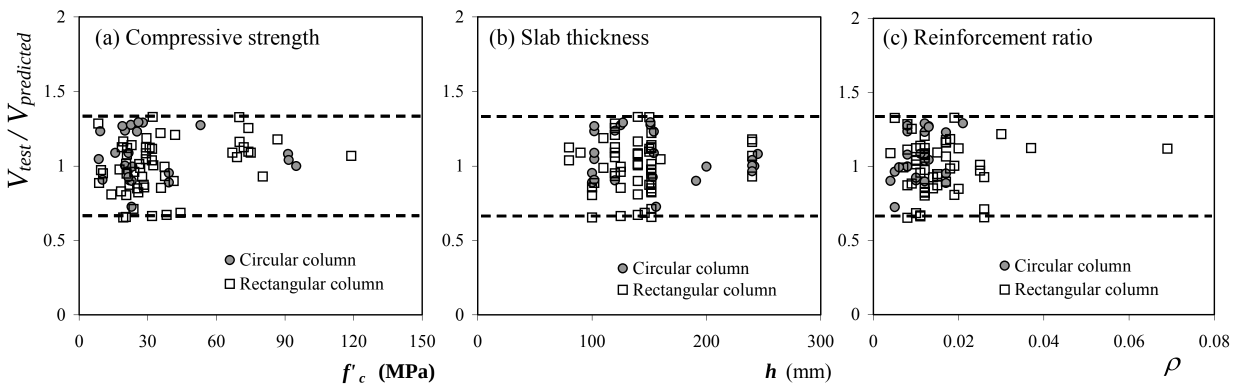


Figure 4: Strength prediction by fuzzy-based model compared with test results

Moreover, observing Figure 2(a), the CEB-FIP MC 90 code underestimates the punching shear strength of specimens with low reinforcement ratio while it overestimates the punching shear strength of specimens with high reinforcement ratio. The Eurocode 2 (2002) shows a good accuracy in predicting the punching shear strength at diferent reinforcement ratios. Finally, the ACI 318 (2005) underestimates the punching shear strength of specimens with high reinforcement ratio while it overestimates the punching shear strength of specimens with low reinforcement ratio. This is attributed to the fact that ACI 318 does not account for the effect of the reinforcement ratio on the punching strength. It is also evident from Figure 4 (c) that the proposed fuzzy-based model predicts shear strength of both low and high reinforcement ratios with consistent accuracy. It is worth noting that the two parameters found to be significant for modeling: the slab thickness and the tension reinforcement ratio have also been promoted by other researchers before because of their influence on the size effect (Bažant 1997) and their possible role in developing shear friction (Loov 1998).

Finally, to make use of the fuzzy-based model in design of slab-column connections and to avoid complexity needed when using all the equations representing the fuzzy-based model, we suggest developing a set of design charts based on predictions by the fuzzy-based model. Figure 5 shows exemplar design charts to evaluate the punching shear strength of slab-column connections using the fuzzy-based model. The desgin charts are developed for a wide range of primary design parameters: $20 \le f'_c \le 100$ (MPa), $150 \le h \le 250$ (mm), and $0.8 \le \rho \le 1.0$ (percent). It can be observed from Figure 5 that the punching shear strength increases with increasing the concrete compressive strength, increasing the tension reinforcement ratio, and decreasing the slab thickness. This reverse thickness effect is known as the size effect (Bažant 1997). This indicates that the fuzzy-based model sucessfully describes the characteristical behaviour of slab-column connections.

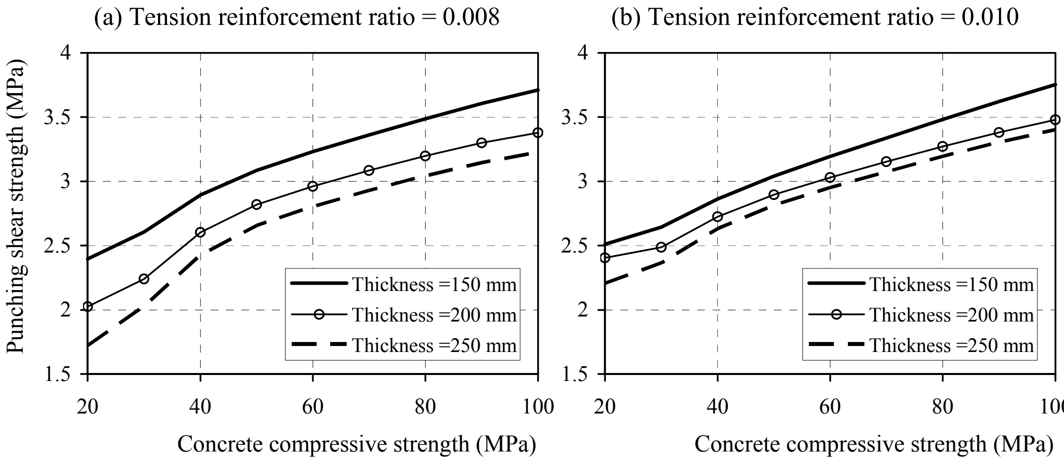


Figure 5: Strength variation according to primary design parameters using fuzzy-based model

CONCLUSIONS

A new alternative method for predicting the punching shear strength of simply supported interior slab-column connections using fuzzy systems is suggested. 176 test specimens were used for training and testing the proposed model (96 training and 80 testing). The training and testing data sets cover a wide range of the material and geometric properties. The testing data set was not used in the training process. Investigations for developing a model with good accuracy showed that concrete compressive strength, slab thickness and tension reinforcement ratio are the primary parameters which dominate the punching behavior of slab-column connections. This finding is limited to circular and rectangular columns with (c_1/c_2) ratio equals to 1.0 and for slabs with perimeter to slab depth ratio (b_0/d) ranging between 5.8 and 20.8. It is found that the punching shear strength predicted by the fuzzy-based model is more accurate than current design codes including CEB-FIP MC 90, Eurocode 2, and ACI 318.

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