STOCHASTIC MODELING OF INFRASTRUCTURE DETERIORATION: AN APPLICATION TO CONCRETE BRIDGE DECKS

George Morcous[1], and Amin Akhnoukh[2]

ABSTRACT

This paper presents an application for the development of stochastic models to predict the deterioration of infrastructure facilities. The main objective is to demonstrate the capabilities and limitations of two types of models, namely state-based and time-based models, which will guide decision makers in selecting the most appropriate model type according to management needs and data availability. Concrete bridge decks are selected for this application because they are considered one of the most deteriorating infrastructure components. Inventory and condition data required for developing the stochastic deterioration models are obtained from the database of the Ministére des Transports du Québec (MTQ). Markov-chain models, as an example of state-based models, and non-parametric time-based models are developed for bridge decks when no maintenance actions are taken. Although this application demonstrates the development of stochastic deterioration models for concrete bridge decks, these models can be developed to predict the performance of other infrastructure facilities for network level analysis.

KEY WORDS

bridge decks, deterioration models, maintenance management, decision making, stochastic approaches

INTRODUCTION

Infrastructure Management Systems (IMSs) have been developed to assist asset managers in maximizing the safety and serviceability of infrastructure facilities within the available budget by making cost-effective maintenance, rehabilitation, and replacement decisions (Hudson et al. 1997). The quality of these decisions depends significantly on the accuracy and efficiency of the deterioration models used to predict the time-dependent performance and remaining service life of infrastructure facilities (Madanat et al. 1997). A deterioration model is defined as a link between measures of facility condition that assess the extent and

---

[1] Assistant Professor, Department of Construction Systems, College of Engineering, University of Nebraska – Lincoln,1110 South 67th Street, PKI 105B,Omaha, NE, 68182-0571, Tel. (402) 554-2544, Fax (402) 554-3304, E-mail: gmorcous@mail.unomaha.edu

[2] Graduate Assistant, Department of Construction Systems, College of Engineering, University of Nebraska - Lincoln, 1110 South 67th Street, PKI 105B,Omaha, NE, 68182-0571, Tel (402) 554 3537, E-mail: aakhnoukh@mail.unomaha.edu

severity of material damages, and vectors of explanatory variables that represent the factors affecting facility deterioration, such as age, material properties, applied loads, environmental conditions, etc. (Ben-Akiva and Gopinath 1995). Several deterministic and stochastic approaches have been developed to model infrastructure deterioration (Morcous et al. 2002). Deterministic approaches, such as straight-line extrapolation, S-shaped curves, and multiple regression, have the advantages of being simple to develop and easy to use. However, the existence of deterioration parameters that are not typically observed or measured, subjectivity and inaccuracy of infrastructure inspection, and stochastic nature of the deterioration process led to the wide spread of stochastic models. These models are able to capture the physical and inherent uncertainty, model uncertainty, and statistical uncertainty while predicting the future performance of infrastructure facilities (Lounis and Mirza 2001).

Although the deterioration of infrastructure facilities is a continuous and gradual process that may span over several decades, discrete ratings or states are commonly used to represent facility conditions. This is because discrete rating systems simplify facility inspection, deterioration modeling, and maintenance optimization (Madanat and Wan Ibrahim 1995). Examples are: the discrete condition rating scale from 0 to 9 adopted by the Federal Highway Administration (FHWA) to evaluate the substructure, superstructure, and deck of highway bridges (FHWA 1995); and the condition rating scale introduced in 1995 within Pontis, the most popular bridge management system in US, using a 5-state scale to evaluate the condition of about 140 standardized bridge elements (Pontis 2005).

Stochastic models used to predict the deterioration of infrastructure facilities can be grouped into two main categories: state-based models and time-based models (Mauch and Madanat 2001). State-based models predict the probability that a facility will have a change in its condition state during a fixed time interval and accumulate this probability over multiple intervals. Markov chain models and semi-Markov models are the most common example of state-based models. Time-based models predict the probability distribution of the time taken by an infrastructure facility to change its current condition state to the next lower condition state. Parametric, semi-parametric, and non-parametric models have been proposed to represent the probability distribution of the transition time.

This paper demonstrates the development of state-based and time-based stochastic deterioration models for reinforced concrete (RC) bridge decks. RC decks were selected because they are one of the most deteriorated infrastructure components due to their direct exposure to traffic loads, environmental degradation factors, frequent freezing and thawing cycles, and using de-icing chemicals in winter. The first section presents the data used in model development. The second and third sections present, respectively, the development of Markov-chain models and non-parametric time-based models for RC decks. The last section discusses the capabilities and limitations of each category of stochastic models as well as recommendations for their applicability in IMSs.

BRIDGE DECK DATA

The condition data used in developing state-based and time-based stochastic deterioration models were obtained from the Ministére des Transports du Québec (MTQ) database, which is a part of a comprehensive system for managing 57 different types of highway structures in Québec, Canada. Reinforced concrete (RC) decks in beam bridges are selected for the model development because beam bridges are considered the most dominant type of structures, since it represents about 60% of the 9678 provincially-owned highway structures. The condition data of RC decks represent the results of the detailed visual inspections carried out approximately every three years. These data comprise two condition ratings (MTQ 1995): (i) Material condition rating (MCR), which represents the condition of a deck based on the severity and extent of observed defects, and (ii) Performance condition rating (PCR), which describes the condition of a deck based on its ability to perform the intended function in the structure. Both the MCR and PCR are represented in an ordinal rating scale that ranges from 1 to 6, where 6 represents the condition of a new and undamaged deck. Because MCR is the governing parameter in most of MTQ maintenance decisions, deterioration models will be developed for MCR only. Figure 1 shows how the MCR of any element is determined given the type of element (i.e., primary, secondary, or auxiliary), percentage of the material defects in the element cross-section, surface area, or length, and the severity of these defects (i.e. very low, low, medium, severe, and very severe).

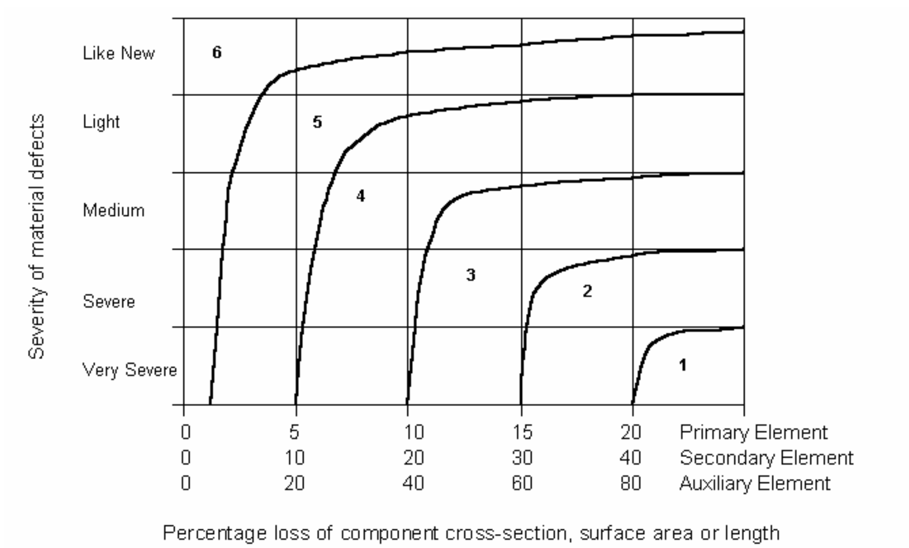

Figure 1: Material condition rating system used by MTQ (MTQ 1995)

To obtain a consistent, complete, and adequate data set for the development of stochastic models, the MTQ databases of years 1997, 1998, 1999, and 2000 were accumulated into one database, which contain inspection records since 1993. This database was screened by filtering out duplicate records and the records with incomplete inspection data (i.e. missing inspection date, or MCR), which resulted in 9181 RC decks (i.e. decks of different spans are considered separately). Each deck consists of seven elements that are evaluated in every inspection: wearing surface, drainage system, two exterior faces, two end portions, and the middle portion. The overall condition of the bridge deck (MCR) is calculated as the aggregation of the MCRs of the seven elements using the balancing factors defined by bridge experts in the MTQ bridge management system (MTQ 1997). RC decks with AC overlay as a wearing surface are selected because they represent 93% of the RC decks in Québec. Other deterioration parameters that affect the performance of RC decks, such as climatic region, highway class, average daily traffic, and percentage of trucks, were overlooked in this study to simplify the development of state-based/time-based stochastic deterioration models. For more information on the effect of these parameters, please refer to Morcous et al. (2003).

STATE-BASED STOCHASTIC DETERIORATION MODELS

State-based models are those used to predict and accumulate the probability of transition from one condition state to another over multiple discrete time intervals (Bogdanoff 1978). These models (sometimes called Markovian cumulative damage models) predict the performance of a network, facility, or component in terms of discrete condition rating using transition probability matrices (TPMs). A TPM *(P)* of order *(n x n)*, where *(n)* is the number of condition ratings, consists of transition probabilities for all possible condition changes over a specific time period given the values of governing deterioration parameters and maintenance actions taken. For the "do-nothing" maintenance action and a short transition period (i.e. one or two years), the elements of a TPM can be assumed to be zeros except for the diagonal line and the line above it., which means that the facility condition will either remain unchanged or drop at most one point in the rating scale. Using the MTQ condition rating system, bridge decks with initial condition vector *P(0)* will have a future condition vector *P(t)* after *(t)* number of transition periods calculated as follows (Collins 1972):

$$P(t) = P(0) * P^{t} \tag{1}$$

where, $$\mathbf{P} = \begin{bmatrix} p_{66} & 1 - p_{66} & 0 & 0 & 0 & 0 \\ 0 & p_{55} & 1 - p_{55} & 0 & 0 & 0 \\ 0 & 0 & p_{44} & 1 - p_{44} & 0 & 0 \\ 0 & 0 & 0 & p_{33} & 1 - p_{33} & 0 \\ 0 & 0 & 0 & 0 & p_{22} & 1 - p_{22} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Conventional first-order Markov-chain models use constant transition probabilities assuming that the future condition of a facility depends only on its initial condition and not on the past condition (i.e. state independence assumption) or even the time elapsed in the initial condition (i.e. stationary process) for simplicity purposes. More realistic models have been developed to account for the effect of the time spent in the initial condition on transition probabilities (i.e. semi-Markov or non-stationary process) and to release the state independence assumption by accounting for the past condition among other explanatory variables (DeStefano and Grivas 1998; Madanat, et al. 1997). Also, several methods have been adopted to estimate transition probabilities, such as percentage prediction method, expected-value method, Poisson regression, negative binomial regression, ordered probit model, and random-effects model (Mauch and Madanat 2001).

Stationary Markov-chain models are developed as an application example of state-based stochastic deterioration models using the expected-value method. In this method, the bridge deck data are plotted on a two dimensional chart, where the horizontal axis represents the age in years, and the vertical axis represents the MCR. The regression model *Y(t)* that best fits the data points is obtained as shown in Figure 2.

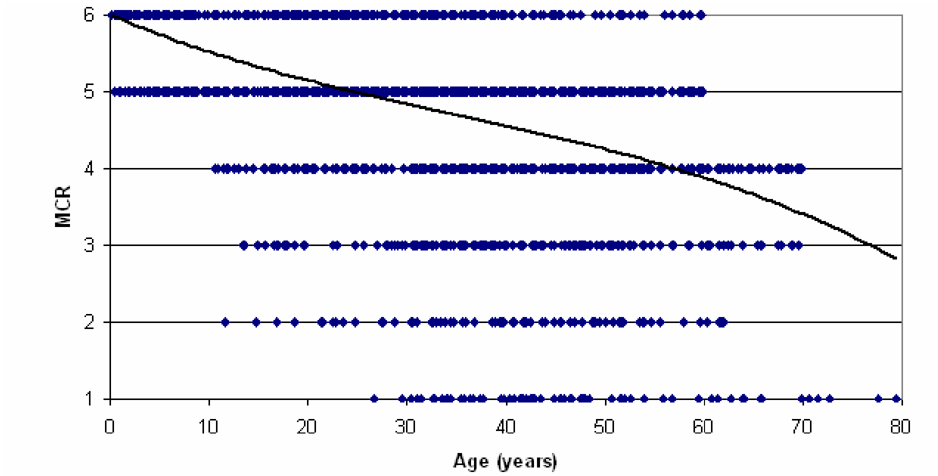

Figure 2: The regression model that best fits bridge deck data

Transition probabilities are then estimated by solving the non-linear optimization problem that minimizes the sum of absolute differences between the regression model *Y(t)* and the expected value *E(t)* predicted using the Markov-chain model as the product of the condition vector *P(t)* and the vector of condition states (Madanat et al. 1995). The objective function and the constraints of this optimization problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{t=1}^{N} \left| Y(t) - E(t) \right| \tag{2}$$

$$\text{Subject to} \quad : \; 0 \le p_{ij} \le 1 \qquad \text{for} \qquad i, j = 1,2,\ldots\ldots, n$$

$$\sum_{i=1}^{n} p_{ij} = 1$$

By solving the above optimization problem, the elements of the one-year TPM shown in Equ. (1) are estimated. This matrix is then raised to the power three to calculate a three-year period TPM as shown below to be used in the condition prediction of MTQ bridge decks.

$$\boldsymbol{P} = \begin{bmatrix} 0.839 & 0.159 & 0.002 & 0.000 & 0.000 & 0.000 \\ 0 & 0.958 & 0.040 & 0.002 & 0.000 & 0.000 \\ 0 & 0 & 0.890 & 0.101 & 0.008 & 0.000 \\ 0 & 0 & 0 & 0.792 & 0.178 & 0.030 \\ 0 & 0 & 0 & 0 & 0.625 & 0.375 \\ 0 & 0 & 0 & 0 & 0 & 1.000 \end{bmatrix} \tag{3}$$

IME-BASED STOCHASTIC DETERIORATION MODELS

Time-based models (sometimes called duration models) are those used to predict probability distributions of facility transition times given the values of governing deterioration parameters, such as design and construction attributes, environmental and operation conditions, and maintenance practice. Transition time is defined as the time needed for a facility to change from an initial condition state to the next lower state in the condition rating scale. The length of the transition time varies significantly from one facility to another due to inherent stochastic nature of the deterioration process and the existence of unobserved/ unmeasured explanatory variables. State transition events recorded in the IMS database are used to perform the life data analysis required to study facility deterioration characteristics and develop cumulative distribution functions of transition time for different condition states. Examples of time-based deterioration models developed using life data analysis are the model developed for deck system using New York State Thruway Authority database (DeStefano and Grivas 1998), and that developed for bridge decks using Indiana Bridge Inventory database (Mauch and Madanat 2001).

The information required for developing time-based stochastic models consists of condition state transition events and the corresponding time data, which will be obtained from the MTQ database. Condition state transition events are identified using sequential changes in MCR of RC decks. Actual time of these changes cannot be easily identified because visual inspections are performed only every three years, in addition, the condition data available to the authors cover only the period from 1993 to 2000. Therefore, adequate sequential condition data can be obtained for only the most common condition states (state 5 and state 4), and their related time data are considered "multiply censored". Censored data means that the observed event (state transition in this study) does not take place during the observation period, however, it is known that the event takes place after a specific time (right censored), before specific time (left censored), or both (interval censored) (Nelson 1982). If this specific time is constant for all data records, it is referred to as "singly censored", otherwise it is referred to as "multiply censored", which is the case of MTQ data.

Life data analysis of the multiply censored data is performed using Kaplan and Meier methods to estimate non-parametric survival and hazard function of RC bridge decks. The survival function *S(t)*, sometimes called reliability function *R(t)*, represents the probability that a bridge deck remains in its condition state for at least time *(t)*. This function can be expressed as follows:

$$S(t) = 1 - F(t) = 1 - \int_0^t f(t)dt \tag{4}$$

where *t* is the random variable that represents the transition time (sometimes referred to as time-in-state), *f(t)* is the probability density function of the transition time *(t)*, and *F(t)* is the corresponding cumulative distribution function. The hazard function *h(t)* represents the instantaneous risk that a bridge deck will change its condition state to the next lower condition state at time t. This function can be expressed as follows:

$$h(t) = \frac{f(t)}{S(t)} \tag{5}$$

Survival and hazard functions developed in this study are considered non-parametric because they do not relate the random variable to any deterioration parameters. Figures 3 and 4 show the survival and hazard functions developed for time-in-state 5 and 4 using the MCR data of RC bridge decks in Quebec. These data contain the sequential condition states (past *i*, current *j*, and future *k*) needed to define transition events. Table 1 lists the different possible condition sequences for a given current condition state *j* and the corresponding method used to calculate the time-in-state (T_j). It should be noted that if the transition event is observed between two consecutive condition states, the transition event is assumed to occur at the middle of the inspection period for simplicity. The data type in this case only is considered complete (not censored), which represents 15% of the data used for estimating time-in-state 5 and 20% of the data used to estimate time-in-state 4. The high percentage of multiply censored data (i.e. 85% and 80%) justifies the development of non-parametric models and the use of Kaplan-Meier Method (DeStefano and Grivas 1998).

Table 1: Sequential condition states and the corresponding time-in-state

Condition States			**Time-in-state**	**Data Type**
i	**j**	**k**	$\mathbf{T_j}$	
j+1	*j*	*j-1*	$D_{ij}/2 + D_{jk}/2$	*Complete*
j+1	*j*	*j*	$D_{ij}/2 + D_{jk}$	*Censored*
j	*j*	*j-1*	$D_{ij} + D_{jk}/2$	*Censored*
j	*j*	*j*	$D_{ij} + D_{jk}$	*Censored*

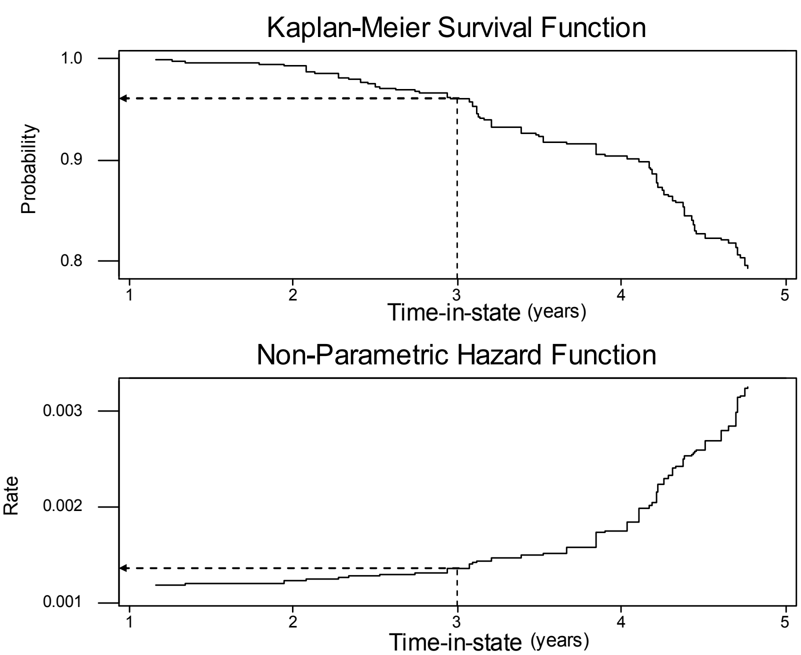

Figure 3: Survival and hazard functions for time-in-state 5

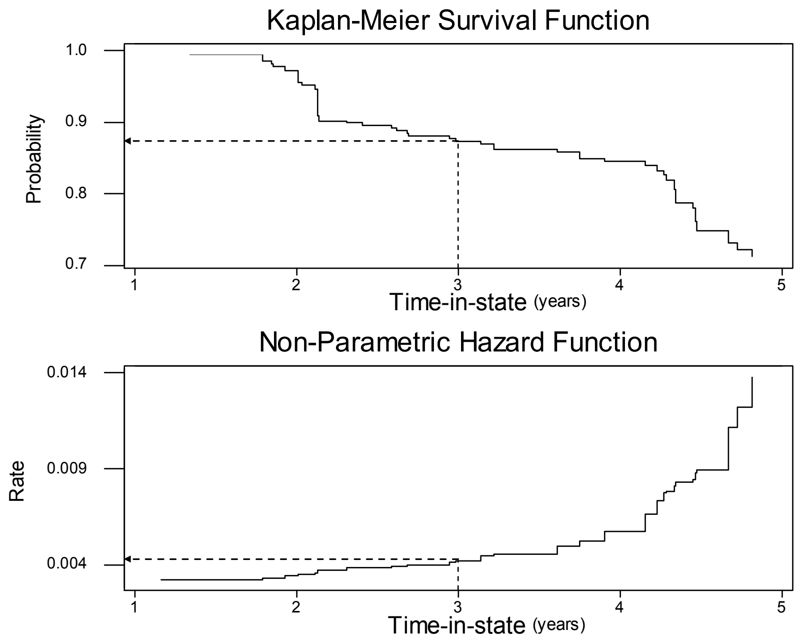

Figure 4: Survival and hazard functions for time-in-state 4

DISCUSSION

The state-based models and time-based models developed using the same data set, as presented in the previous section, are highly related (Mauch and Madanat 2001). The information obtained from a state-based model can be used to develop/validate the corresponding time-based model and vise versa. For example, if the probability distribution of the time-in-state is known, the transition probability from one condition state to another can be easily estimated for a given transition period. Also, if several transition probabilities between two specific condition states are available for different transition periods, the probability density function of the transition time can be estimated. These relationships can be verified by finding the probabilities from Figures 3 and 4 that correspond to a time-in-state equals to the transition period of the developed state-based model (indicated by dotted lines). These probabilities were found to be approximately 96% and 88%, which are almost similar to the elements (p_{55}) and (p_{44}) in the transition probability matrix shown in Equ. (3), respectively. In addition, the time-based models can be used to release the stationary process assumption of the conventional Markov-chain models by estimating the transition probabilities as functions of the time elapsed in the initial condition. It should also be noted that the decision of which type of models is more appropriate for deterioration prediction is highly dependent on the nature of the available condition data (Mishalani and Madanat 2002). Frequent inspections over a long observation period are required for developing time-based models, while infrequent inspection over a relatively short observation period can be used for developing state-based models.

SUMMARY

This paper demonstrated the development of two different types of stochastic models for predicting the deterioration of concrete bridge decks. A Stationary Markov-chain model was developed as an example of state-based stochastic models. Bridge deck data, such as age and material condition rating (MCR), were obtained from the Ministére des Transports du Québec database. The expected-value method was used to generate transition probability matrix for the "do-nothing" maintenance action. Life data analysis was used to estimate non-parametric survival and hazard functions required for developing time-based stochastic models. Kaplan-Meier method was adopted for this analysis because of the high percentage of multiply censored records in the available condition data. The probability distributions of time-in-state 5 and 4 were calculated using the developed functions. This application demonstrated the relationship between the two different types of stochastic models and the level of detail that can be obtained when each types is adopted. The decision of which type of models should be used in an IMS is highly dependent on the data availability and the degree of accuracy required by decision makers for network-level analysis.

ACKNOWLEDGEMENT

The authors wish to acknowledge M. Guy Richard, Eng., Director, and M. René Gagnon, Bridge Engineer, of the Structures Department - Ministére des Transports du Québec - for their invaluable help in providing the authors with all available data, manuals and other needed information.

REFERENCES

Ben-Akiva, M., and Gopinath, D. (1995) "Modeling Infrastructure Performance and User Costs", Journal of Infrastructure Systems, ASCE, 1(1), 33-43.

Bogdanoff, J.L. (1978). "A new cumulative damage model- Part 1." Journal of Applied Mechanics, 45, 246-250.

Collins, L. (1972) "An Introduction to Markov Chain Analysis", CATMOG, Geo Abstracts Ltd., University of East Anglia, Norwich.

DeStefano, P. D, and Grivas, D. A. (1998) "Method for Estimating Transition Probability in Bridge Deterioration Models", Journal of Infrastructure Systems, ASCE, 4(2), 56-62.

Federal Highway Administration (FHWA) (1995) "Recording and Coding Guide for the Structure Inventory and Appraisal of the Nation's Bridges", US Department of Transportation, Washington, DC.

Hudson, W. R., Hass, R., and Uddin, Waheed (1997) "Infrastructure Management", McGraw-Hill, NY.

Lounis, Z., and Mirza, M. S. (2001) "Reliability-Based Service Life Prediction of Deteriorating Concrete Structures", Proc. 3rd Int. Conf. on Concrete Under Severe Conditions, Vancouver, June.

Madanat, S. and Wan Ibrahim, W.H., (1995) "Poisson Regression Models Of Infrastructure Facility Deterioration", Journal of Transportation Engineering, ASCE, 121(3), 267-272.

Madanat, S., Karlaftis, M.G., and McCarthy, P.S., (1997), "Probabilistic Infrastructure Deterioration Models with Panel Data", Journal of Infrastructure Systems, ASCE, 3(1), 4-9.

Madanat, S., Mishalani, R. and Wan Ibrahim, W.H., (1995), "Estimation of Infrastructure Transition Probabilities From Condition Rating Data", Journal of Infrastructure Systems, ASCE, 1(3), 120-125.

Mauch, M., and Madanat, S., (2001) "Semiparametric Hazard Rate Models of Reinforced Concrete Bridge Deck Deterioration" Journal of Infrastructure Systems, ASCE, 7(2), 49-57.

Ministére des Transports du Québec (MTQ) (1995) "Manuel d'Inspection des Structures: Evaluation des Dommages", Bibliothèque Nationale du Québec, Gouvernement du Québec, Canada.

Ministére des Transports du Québec (MTQ) (1997) "Manuel de l'Usage du Système de Gestion des Structures SGS-5016", Bibliothèque Nationale du Québec, Gouvernement du Québec, Canada.

Mishalani, R., and Madanat, S., (2002) "Computation of Infrastructure Transition Probabilities Using Stochastic Duration Models", Journal of Infrastructure Systems, ASCE, 7(2), 49-57.

Morcous, G., Lounis, Z., and Mirza, S. M. (2003) "Identification of Environmental Categories for Markovian Deterioration Models of Bridge Decks", Journal of Bridge Engineering, ASCE, 8(6), 353-361

Morcous, G., Rivard, H., and Hanna, A. M. (2002) "Modeling Bridge Deterioration Using Case-Based Reasoning", Journal of Infrastructure Systems, ASCE, 8 (3), 86-95.

Nelson, W. (1982) "Applied Life Data Analysis", John Wiley & Sons, NY.

Pontis (2005), Release 4.4 User's Manual, American Association of State Highway and Transportation Officials (AASHTO), Cambridge Systematic Inc.