COMPUTING THE DESIGN NUMBER OF LANES ON FREEWAYS BY INTEGRATING THEORY, GEOMETRY, AND VEHICLE CHARACTERISTICS

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ABSTRACT

The number of lanes required for freeway design on an uphill road section is examined on a unified computational framework by integrating together vehicle characteristics, road geometry, traffic flow theory, and traffic compositions. Four important parameters are introduced to characterize the formation and propagation of queues initiated by slow trucks on uphill road sections. Important design criteria are derived and computed on physical bases to show the interesting interrelation among these factors on freeway design. Typical examples are presented to illustrate the proposed computational framework in contrast to the conventional strategies recommended by the HCM guideline. These criteria should be considered carefully by engineers to avoid recurrent traffic congestion initiated at certain locations of freeways where an uphill road section is to be constructed, rehabilitated, or widened for smoothing traffic operations. Insufficient number of lanes in geometric design can seed a network with many 'congestion' sources, which can initiate traffic jams in peak traffic periods or when future traffic demand or volume thrives.

KEY WORDS

Capacity, Criteria, Curve, Density, Geometry, Grade, HCM, Lane, Length, Power, Queue, and Speed.

INTRODUCTION

Driving in large metropolitan areas is no more enjoyable. In addition to confronting with numerous confusing traffic signs along freeways and streets, one may experience traffic queues and traffic jams due to the presence of one or several slow moving vehicles, a rubber neck, and occasionally a vehicle accident blocking a lane or two. Slow moving vehicles are usually trucks or other heavy vehicles. They move slowly uphill when loaded, serving as possible sources for introducing queues and jams into a traffic network. This cause phantom bottlenecks when traffic becomes relatively heavy [Gazis & Herman, 1992]. This phantom

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bottleneck was further considered transient on flat freeways if it dissipates [Newell, 1998]. The phantom bottleneck becomes recurrent when flow rate is high and traffic is mixed well with enough number of heavy vehicles. The bottleneck problem arising from an uphill section can be a serious problem on highway design. This problem may be solved by introducing design strategies and incorporating proper transportation planning, traffic flow characteristics and traffic composition analysis.

In this paper, formation of queues or local congestion on uphill road sections is considered by computing the queue indicators γ_m and γ_{ns} respectively for steady and non-steady flow conditions. In addition, the propagation of the queues, due to the presence of trucks in either a steady or a non-steady traffic stream, is characterized respectively by the queue growth indicators Γ_m and/or Γ_{ns}. The calculation of these parameters is carried out by taking into account the road geometry, flow conditions in traffic streams, traffic compositions, and vehicle characteristics. Interesting functional relationships relating these physical variables to the parameters are derived. These relations or criteria are then applied to determine the number of freeway lanes needed to accommodate daily traffic in some interesting scenarios. The computational results are then compared with the results found by following the HCM. .

FORMULATION

Consider an uphill road section of length L and a positive grade G, and a loaded truck with weight W is climbing the section. The rolling friction coefficient on the tir-road contact is given by approximately

$$f = 0.01 \times (1 + \beta \, \mathrm{v}), \qquad (1)$$

where the parameter β is approximately equal to 0.0223 s/m [Mannering & Kilareski, 1990]. One may estimate the climbing speed v of a loaded truck with P horsepower by equating the product and the speed and the frictional forces to the power, yielding

$$100(1+\tilde{G}^2)^{1/2}\zeta P/W = (1 + \mathrm{G} + \beta \, \mathrm{v})\mathrm{v} \qquad (2)$$

where quantity $\tilde{G} = G/100$; and quantity ζ is the parameter reflecting the efficiency of a vehicle engine, ranging from 0 to 0.9 [Mannering and Kilareski, 1990]. Where the solution of speed v_1 for Equation (2) is found to be

$$\mathrm{v}_1 = 2\alpha \times \left[(1+G) + \sqrt{(1+G)^2 + 4\alpha\beta} \right]^{-1} \qquad (3)$$

where parameter $\alpha = 100(1+\tilde{G}^2)^{1/2}\zeta P/W$. It can be inferred from Equation (3) the speed drops approximately inverse proportional to the grade G if G becomes 'large'. The capacity per lane due to the presence of a loaded truck on a road section of a positive slope can be computed using

$$\mathrm{q} = \mathrm{kv} = \mathrm{v_f}\,\mathrm{k}\exp(-\lambda\,\mathrm{k}^2) \qquad (4)$$

The speed v and concentration k is in units of km/hr and km^{-1}. It can be derived from Eq. (4) that the optimal (maximal) flow capacity is given by $\mathrm{q_{op}} = \mathrm{v_f}(2\lambda e)^{-1/2}$ and the corresponding optimal density is $k_{op} = (2\lambda)^{-1/2}$ (Gazis & Herman, 1992). The flow rate q in the lane taken by a slow truck is determined by the speed of the truck. The steady speed v_1 of a fully loaded truck moving uphill isn't always attained in few seconds because of the low acceleration power of the truck. The fast drop of the road flow rate in the presence of a heavy vehicle can be inferred from the Eq. (4). A slow moving truck acts as a moving bottleneck. The main concern is that whether the effect spreads over the upstream or remains localized in the uphill portion. In the rest of the paper, we are trying to show the reader that when 'macroscopic' flow conditions are right, the spread will occur; but it may remain localized when traffic nearby the uphill section is dilute enough to allow drivers to freely switch lanes.

Consider a two-lane unidirectional road with an uphill section of length L. Denoting the upstream concentration of upstream of the road as k_0, the concentration of the blocked lane due to the presence of a loaded truck as k_1, and the down stream of the 'unblocked' lane as k_2. The corresponding speeds at these regions are v_i (i=0,1,2). The 'unblocked' lane is very persistent because of the lane switching due to impatient drivers in the blocked lane and the presence of trucks in the traffic stream. Applying the number conservation law, one can find the rate of queue growth R behind the moving truck (Gazis and Herman, 1992):

$$R = [k_0(v_0 - v_1) - k_2(v_2 - v_1)/2]/[k_1 - k_0] \qquad (5)$$

Equation (5a) is the direct consequence of the vehicle number conservation [Prigogine & Herman, 1971]. The spill or the outflow of the moving bottleneck is represented by the second term in the numerator. The out flow speed $v = v_2 - v_1$ is always greater than zero and is of the order of 10-20 km/hr. It is a parameter depending on the traffic composition, driver behavior, vehicle characteristics, and other factors. The queue length Q built up when a loaded truck moves from the foot of a hill to the top of the hill is given by

$$Q/L = \gamma = [k_0(v_0 - v_1) - k_2 v/2]/[v_1(k_1 - k_0)] \qquad (6)$$

When quantity $\gamma > 1$, the queue spreads beyond the inclined section of the road; and the queue will continue to propagate upstream if another loaded truck joins the queue, depending on traffic compositions. Let's assume the truck population is θ percent of the total traffic and η percent (>50%) of trucks moves on the blocked lane. Within a time duration L/v_1, the number of trucks arrives at a queue on average before it dissipates will be in the order of $2\eta\theta k_0 v_0 L/v_1$. It becomes physically clear that if the following two conditions are satisfied simultaneously, namely;

$$\gamma > 1 \qquad (7a)$$

$$\Gamma = 2\eta\theta k_0 v_0 L/v_1 > 1 \qquad (7b)$$

A queue will initiate and propagate.

NON-STEADY STATE FLOW CONDITIONS

In deriving equations (5-7), we have implicitly assumed that traffic flow is steady. There are situations in which the flow conditions can be worsened. For example, when a queue grows beyond the length of an inclined road section, heavy vehicles arrive at the queue may be forced to slow down to a very low speed before moving uphill. Then, it takes a 'long' time for a heavy vehicle to reach the desired steady speed v_1, depending on the acceleration power of the vehicle and the road geometry. For a loaded truck, the acceleration power can be small in the neighborhood of $0.3\,\mathrm{m/s}^2$ even at low speed [Wright & Dixon, 2004]. In such case, one may estimate the queue length by considering two scenarios respectively. The first is when the truck finishes climbing uphill before the steady state speed v_1 is reached; and the other is the steady state speed v_1 is reached somewhere in the uphill section. Denoting the queue length ratio Q/L as γ_{ns}, the accerleartion power of the truck as $\hat{\alpha} g$, we can express it in terms of the vehicle and the road geometry parameters.

$$\gamma_{ns} = \begin{cases} [\mathrm{k}_0(\mathrm{v}_0-\mathrm{v}_1)-\mathrm{k}_2 v/2](1-L_1/L)/[\mathrm{v}_1(\mathrm{k}_1-\mathrm{k}_0)] & \\ +(g\overline{\alpha}k_0 L_1/\phi \mathrm{v}_1 k_{op} L)\,\Phi(\alpha,\beta,\mathrm{v}_0,\mathrm{v}_{f,}\ \mathrm{v}_1) & \text{if} \quad \mathrm{v_e}=\mathrm{v}_1 \\ \overline{\alpha}k_0/\phi \mathrm{v}_e k_{op}\Phi(\alpha,\beta,\mathrm{v}_0,\mathrm{v}_{f,}\ \mathrm{v}_e) & \text{if} \quad \mathrm{v_e}<\mathrm{v}_1 \end{cases} \tag{8}$$

Where $\mathrm{v_e}$ is the final speed of the truck moving uphill, and

$$\mathrm{t}_1 = \frac{100}{g\beta}\times \ln[\overline{\alpha}/(\overline{\alpha}-\beta\,\mathrm{v}_1)],\ \overline{\alpha}=100\hat{\alpha}-1-G/\sqrt{1+10^{-4}G^2}$$

$$L_1 = \frac{100\mathrm{v_a}}{g\beta}\times\left[-\mathrm{v}_1/\mathrm{v_a}-\ln(1-\mathrm{v}_1/\mathrm{v_a})\right],\ \mathrm{v_e}<\mathrm{v_a}=\overline{\alpha}/\beta$$

$$\Phi = \int_0^1 \frac{(\tilde{\mathrm{v}}_0-x)-\mathrm{k}_2\tilde{v}/2\mathrm{k}_0}{\tilde{\alpha}\text{ - }\beta x}\times\frac{\mathrm{d}x}{\sqrt{2\ln(\tilde{\mathrm{v}}_\mathrm{f}/x)}-\sqrt{2\ln(\tilde{\mathrm{v}}_\mathrm{f}/\tilde{\mathrm{v}}_0)}}$$

The condition parameter Γ_{ns} for more than one heavy vehicle to join the queue can be modified from Eq. (7b) to the following form:

$$\Gamma_{ns} = \begin{cases} 2\eta\theta\,\mathrm{k}_0\mathrm{v}_0[\tau_1+(L-L_1)/\mathrm{v}_1] & \text{if } \mathrm{v_e}=\mathrm{v}_1 \\ 2\eta\theta\,\mathrm{k}_0\mathrm{v}_0\tau_e & \text{if } \mathrm{v_e}<\mathrm{v}_1 \end{cases} \tag{9}$$

If both parameters Γ_{ns} and γ_{ns} are greater than 1, then the queue behind the slow moving vehicle will grow indefinitely so long as the traffic compositions and the flow rate do not fluctuate. Otherwise, the temporary 'phantoms' of bottlenecks will appear and dissipate when traffic flow is relatively dilute or fluctuates at some periods. One may easily notice that for a sufficiently long uphill road section ($L >> L_1$), both parameters Γ_{ns} and γ_{ns} can be written in terms of their counterparts for a steady state flow and a contribution term due to the presence of a slow truck at the foot of the inclined section, namely:

$$\Gamma_{ns} = \Gamma + 2\eta\theta\, \mathrm{k}_0 \mathrm{v}_0 \tau_1 \tag{10}$$
$$\gamma_{ns} = \gamma + (g\bar{\alpha}k_0 L_1 / \phi \mathrm{v}_1 k_{op} L)\, \Phi(\hat{\alpha}, \beta, \mathrm{v}_0, \mathrm{v}_{f,}\ \mathrm{v}_1)$$

Equations (8)-(10) can be easily programmed to an Excel worksheet to calculate parameters Γ_{ns} and γ_{ns} for different traffic and geometric conditions.

DISCUSSIONS & EXAMPLES

A queue, once initiated, will spread upstream and can potentially clog traffic at interchanges nearby the inclined section. The first condition (7a) initiates a queue beyond the uphill section; and the second condition (7b) if satisfied will attach one or more trucks to the initiated queue and propagate it upstream. The queue then will grow with time on average according to $\sim(\gamma - 1)T\mathrm{v}_1$ under a steady flow condition; and its detail growing characteristics depend on the probabilistic process of truck arrivals. Assuming a truck is moving 48 km/hr (30 mph), and γ is 1.20, a queue will grow to 1.6 km in 6 minutes, which is long enough to affect traffic operation at nearby interchanges especially in an urban freeway system where interchanges are densely distributed. This long-range effect can be a main mechanism triggering network level congestion, which once formed takes long time to dissipate.

When heavy vehicle percentage is relatively high in a traffic stream, parameter γ will reach its maximum value:

$$\gamma_m = [\mathrm{k}_0(\mathrm{v}_0 - \mathrm{v}_1)]/[\mathrm{v}_1(\mathrm{k}_1 - \mathrm{k}_0)] \tag{11}$$

The second term in the numerator of Eq. (6) becomes small compared to the first term when another truck gets on the 'unblocked' lane, leading to a 'locked' situation. Using Eq. (4), we can express Eq. (11) in terms of the flow concentrations:

$$\gamma_m = \{\exp[\lambda \mathrm{k}_0^2\, (\mathrm{k}_1^2/\mathrm{k}_0^2 - 1)] - 1\} / (\mathrm{k}_1 / \mathrm{k}_0 - 1) \tag{11a}$$

Assuming parameter $\zeta = 0.9$ and flow concentration $\mathrm{k}_0 = \mathrm{k}_{op}$=35 vplpkm (corresponding to the flow of $\mathrm{q}_0 = 2450$ pcplph in a level of service E) and computing γ_m in an Excel worksheet, one finds that the design parameter γ_m in table 1 for different accelartaing power of the truck and for different uphill grades

Table 1: Queue Indicator γ_m

	Grade	3%	4%	5%	6%	7%	8%
ζ	0.7	1.15	1.38	1.57	1.76	1.95	2.14
ζ	0.8	1.27	1.50	1.73	1.94	2.15	2.35
ζ	0.9	1.43	1.68	1.93	2.16	1.39	2.61

If a spill velocity $v = 8$ km/hr is used, the quantity γ is found to be .60, 1.01, 1.25, 1.44, 1.62, and 1.80 for the slope of 3%, 4%, 5%, 6%, 7%, and 8% respectively. Although γ varies when the spill speed changes, once a long queue is triggered when $\gamma_m > 1$ or $\gamma > 1$, it will be very difficult to dissipate the queue when the truck composition is 'high', leading to a persistent 'Phantom' bottleneck [Gazis & Herman, 1992]. The threshold θ_c of the truck percentage in a traffic stream beyond which an initiated queue is going to propagate upstream is given by rewriting Eq. (7b):

$$\theta_c = \exp[-\lambda \mathrm{k}_0^2 \, (\mathrm{k}_1^2/\mathrm{k}_0^2 - 1)]/(2\eta L \mathrm{k}_0) \tag{11b}$$

Again consider a flow at optimal density $\mathrm{k}_0 = \mathrm{k}_{\mathrm{op}}$, and an inclined section of length L of 400 meters. Setting parameter $\eta = 75\%$, one finds that the threshold value of θ_c is 4.1%, 3.4%, 2.9%, 2.5%, 2.2%, and 2.0% respectively for a positive grade G of 3%, 4%, 5%, 6%, 7%, and 8%. Note that an uphill road curve of length of 400 m isn't rare in urban areas. The reduction in flow in these situations will be approximated by:

$$\mu = 1 - \exp[-\lambda \mathrm{k}_0^2 \, (\mathrm{k}_1^2/\mathrm{k}_0^2 - 1)]\mathrm{k}_1 / \mathrm{k}_0 \tag{12}$$

In Table 2, both the numerical reduction μ at the threshold value θ_c for a steady flow were calculated using Eq (12) and were compared to reduction factors suggested by HCM [2000], respectively.

Table 2: Capacity Reduction Factor for a 400m uphill section

Grade	3.0%	4.0%	5.0%	6.0%	7.0%	8.0%
θc	4.1%	3.4%	2.9%	2.4%	2.2%	2.0%
Eq (12)	2.0%	7.5%	14.2%	20.5%	26.3%	31.2%
HCM	0.0%	9.4%	10.5%	11.9%	13.8%	16.0%

One may infer from Table 2 that capacity reduction suggested by HCM underestimate the geometric effect of roads. Moreover, the rationale for deriving the capacity reduction suggested by HCM isn't clear, indicating that the validity of the heavy vehicle factor f_{hv} for different grades suggested in HCM owes a physical explanation [TRB 2000]. Examining the geometric factors and the traffic compositions are crucial in selecting freeway design strategies in order to avoid recurrent daily congestion, especially in peak periods. Carefully examining the conditions imposed by Eqs (7a-b) for steady flow and Eq. (8-9) for non-steady flow when performing a study to select design alternatives will help to relieve traffic congestion and improve a network mobility and accessibility to general public.

One may apply the above formula for deciding the number of lanes needed for freeways. Let's assume that (1) an upward 6% grade section of road has a length of 500m; (2) the truck composition in traffic is forecasted to be 5%; (3) the traffic flow volume is 3900 pcph one-way in peak hours; (4) 75 % of trucks moves in the outside lane; and (5) the engine efficiency ζ is 0.9. Following the design guide [TRB, 2000], a two-lane freeway along one direction for this section will provide sufficient capacity. However, by applying Eq. (11a) and Eq. (11b), one finds that γ_m is 1.20 and θ_c is 4.0%, respectively. Note that the critical value θ_c is below the forecasted value of 5%. Thus, a 6-lane freeway is needed in this case over the section of the design curve if similar vertical curves appear in both flow directions. Let us examine another example for rehabilitating a 3-lane freeway section (unidirectional) with a positive grade of 4.0%. Consider that (1) the flow rate is 4400 pcph; (2) the heavy vehicles represent 4% of flow volume; (3) the road section with positive grade is 400 m in length; and 75% of heavy vehicles moves in the outside lane. One may tend to think that keeping 2 lanes open could be sufficient for rehabilitating the outside lane. Using Eqs. (7a) & (7b) and assuming the upstream traffic is well channeled to two lanes before moving uphill, one obtains that $\gamma \approx 1.36$ and $\Gamma \approx 1.20$. The growth rate of the queue toward upstream is about 9 km/hr, indicating that the queue will accumulate to a length of about 1 mile in 10 minutes. Thus, in order to relieve the congestion, an auxiliary lane may be built or the shoulder should be made use for passenger vehicles.

In Table 3, we examine the parameters Γ and γ for various positive grades in two different situations for comparison. One is for steady traffic flow; and the other is for non-steady flow. Let's assume the length of the uphill road section is 300 meters, the upstream is at the optimal flow condition; and the spill velocity of the moving bottleneck ν is assumed to be to zero. The vehicle parameters $\hat{\alpha}$, β, ζ are set to $0.1\text{m}/\text{s}^2$, $0.0223\,\text{s/m}$, and 0.9 respectively. The truck traffic composition θ and the distribution factor η are assumed to be 4.5% and 75%.

Table 3: Queue and Queue Propagation Indicators

Grade	3%	4%	5%	6%	7%	8%
γ_m	1.15	1.38	1.57	1.76	1.95	2.14
γ_{ns}	1.89	1.90	2.08	2.38	2.73	5.44
Γ_m	0.82	1.00	1.17	1.35	1.53	1.71
Γ_{ns}	1.29	1.35	1.45	1.62	1.99	3.15

From the table, one can infer that the flow conditions can be exasperated when a slow truck is present at the foot of an uphill section. This is expected when flow is relatively heavy because there is no enough headway between vehicles for a loaded truck to accelerate to a higher speed before running uphill. The heavy truck is forced to move in a jerking fashion. It takes a longer time duration for the truckt o reach the steady state speed v_1. Thus, a queue longer than the one under a steady state condition form. The situation is worsened when one or more loaded vehicles join the queue before the lead truck picks up the speed.

A city network could be in an alarming situation when sufficient numbers of the uphill freeway sections were present. In a large metropolitan area in US, there are on average two freeways going north/south direction, two freeways going east/west direction, and one ring belt around the metropolitan area. One may find extreme examples such as LA & New York, where freeways are 'numerous'. The freeways excluding the belt way usually 'intersect' at a small region somewhere close to the center of a city. Around the central area of a city, any design curves with positive grade higher than 3% may not be desirable unless enough lanes are provided. An uphill freeway section might has impact on traffic beyond the section. Assuming the acceleration of a heavy truck is $0.5\,\mathrm{m/s}^2$, it takes about 500 meters for a loaded truck to resume its normal speed. Once the truck is slowed down, it takes a 'long' time for the vehicle to achieve the normal speed around 96 kph; and the longer it takes, the longer a queue will accumulate behind the heavy vehicles. Bottelnecks initiated by a truck may propagate miles upstream and clog interchanges, which feed high volume of traffic from local arterials/highways to freeways in peak periods. This clogging can potentially touch off congestion in an urban network, causing higher fuel consumption, heavier vehicle emissions, longer traveling time, unpleasant driving conditions, and possibly higher accident rates in freeways. In general, within the urban area surrounded by a beltway, if a long vertical curve is to be constructed, the design must be executed carefully by providing a sufficient number of lanes over the length of the curve. The consequences could be disastrous if several spots in the freeway network of a city were seeded with long vertical curves of insufficient numbers of lanes. Once bottolneck effects from various spots overlap, the congestion would become global and would initiate a network-wide jam. In order to avoid congestion at the network level, on average, the number of the trouble spots Z should be limited by

$$Z \approx \sqrt{A} / \langle (\gamma_m - \mathbf{1}) v_1 \rangle \qquad (13)$$

The parentheses in the denominator indicate the queue length within the brackets is taken as the average. Assuming the area of a city is 1024 square kilometers, the average queue in the peak hours caused by a trouble spot is about 5 km, and there should not be more than 6 such troubled spots in the network. The main purpose of applying Eq. (13) in design is to stop globalization of localized congestion initiated by slow trucks moving on uphill road sections in a network.

CONCLUSIONS

The impact of an uphill road section or the uphill portion of vertcical curves on traffic flow is computed using Excel worksheets by considering the vehicle characteristics, the grade of the curve, the flow conditions, traffic composition in the flow stream. The computational results are shown in several tables. The existence of large vehicles in a traffic stream can reduce the capacity of the freeways in the positive grade section of a vertical curve dramatically if traffic becomes heavy. This makes a queue behind a heavy vehicle to grow indefinitely. The queue can become long enough to block upstream interchanges when the percentage of the heavy vehicles in a traffic stream reaches certain limit. Two criteria for a queue to grow beyond a road section with a positive grade are derived and applied to determine number of lanes needed for a few cases in both design and rehabilitation in contrast to the unexplained design method following the HCM manual. Particular attention should be paid to the non-steady flow conditions, which are very likely to occur when truck composition is high, the grade of a road section is beyond 3%, and the length of the road section is long. These macroscopic flow conditions are condensed to the formula for the queue indicator γ_{ns} or γ_m and queue propagation indicator Γ_{ns} or Γ_m. When designing an uphill road section, one should first compute the indicators γ_m and Γ_m, if these parameters show that a queue would form, one needs to add an auxiliary lane for maintaining a smooth flow in peak periods. If parameters γ_m and Γ_m are not well below 1, one must compute indicators γ_{ns} and Γ_{ns} to make sure that both indictors are less than 1. Or else, an auxiliary or extra lane is needed to avoid the initiation of traffic jams in peak hours. The criteria are of great importance in design & rehabilitation for avoiding recurrent congestion in urban networks. They can be further applied to highway design in areas where rapid future development is foreseeable and anticipated.

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