# Application of reliability-based system assessment using a bridge example

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ABSTRACT: Structural health monitoring during the life-cycle of a structure is necessary to ensure permanently the bearing capability and the serviceability respectively. Differences from the designed properties can be assessed consequently and therefore rehabilitation measurements can start. For the minimisation of the structural health monitoring measures and the cost involved, the monitoring measures must be concentrated on the critical weak points of the structure. Therefore the knowledge-based system PROBILAS (PRObabilistic Building Inspection and Life ASsessment) is developed. By the combination of recognized procedures of reliability and system analysis in PROBILAS a continuous revaluation of the building and the identification of the failure-relevant parts is possible. This article illustrates the building assessment process with PROBILAS using a bridge as an example. The process includes repeated evaluation of the system and the focusing of both the stochastic an the physical models on the failure-relevant parts of the system.

#### 1 INTRODUCTION

#### 1.1 Motivation

The life time oriented design of structures includes maintenance strategies. Structural health monitoring is essential to evaluate these strategies in such way that the bearing capacity, serviceability and the durability remain ensured and the costs of rehabilitation are limited.

The aim of structural health monitoring is the continuous monitoring and assessment of the present state of the structure. The outcome of this is the base for the optimisation of further measures.

The main focus of the collaborative research center (CRC) "Life cycle assessment via innovative monitoring" funded by the DFG at Braunschweig University of Technology is to optimize methods of structural monitoring. In the following a part of the work of project field A1 of the CRC for the reliability based system assessment is described. The developed methods are able to identify critical weak points and failure paths. The monitoring measures will be concentrated on these points. This leads to a maximum benefit with respect to safety and information with limited investments.

# 1.2 Reliability-based system assessment

The first step in reliability-based assessment is to acquire all necessary input data describing the structure. In the next steps the structure is analysed with

methods of system theory. By combining these methods with methods of reliability theory, a describing model of the structure is created. The reliability analysis starts with the identification of the different sources of risk of the structure. Afterwards, the system has to be discretised in causally connected components and subsystems. These relations are summarized in a fault tree. The fault tree regards all possible causal sequences of component and subsystem failures that lead to system failure. For all components in a fault tree a limit-state has to be defined. The limit state is described by quantities that represent the resistance of a structure (e.g. material strength) and by quantities, which represent the actions imposed on the structure (e.g. live load).

For the reliability analysis the first/second order-reliability method (FORM/SORM) (Ditlevsen & Madsen 2003) is utilized. In these calculations a probability of failure ( $p_f$ ), a safety index  $\beta$  ( $p_f$ = $\Phi$ (- $\beta$ )) and sensitivity values ( $\alpha$ ) for each parameter can be calculated from the limit state equation. For the calculation of the system reliability the computer code STRUREL (RCP 2004) is used. With the help of the system reliability analysis the weak points ("hot spots") of the structure can be identified. On the basis of the calculated values, the failure path with the highest probability of occurrence can be found. Especially the parameters of the limit-state equations within this failure path should be investigated further.



These methods are implemented in the knowledge-based system PROBILAS (PRObabilistic Building Inspection and Life ASsessment) (Klinzmann & Hosser 2005, Hosser et al. 2004).

# 2 APPLICATION OF THE RELIABILITY BASED SYSTEM ASSESSENT

The function of a building regarding the bearing capacity and serviceability is to be guaranteed over the intended service life without substantial loss of the usage characteristics. The necessary measures for reconditioning of a building may not become inadequately large at this. The detection of safety-relevant deviations from planned properties requires an optimized structural health monitoring. Consequently the service life is ensured with a minimum of rehabilitation. The main focus in this article lies on the ultimate limit-states of the bridge structure with consideration of a corrosive damage of the tendons. The structure is a single span plate girder bridge with two girders and a span width of 25 m (Fig 1, 2).

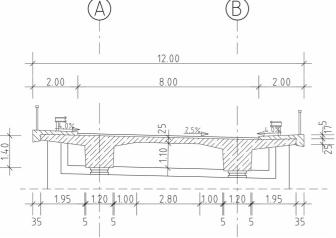


Figure 2. Bridge example, section

The bridge is pre-stressed with post-tensioning tendons. The width of the driving lane is 10.5 m, since

the bridge is dimensioned for short-distance traffic.

#### 3 PROBABILISTIC MODEL

#### 3.1 Fault tree

The first step is the development of the fault tree with the assistant of the knowledge-based system PROBILAS. After the classification of the type of building and the used structural materials, typical weak points can entered. These weak points should be considered in the fault tree. Failure modes like shear failure at the support or flexure failure at midspan for example are weak points to be inspected. These obvious examples are as well part of the structural analysis. Furthermore environmental influences have to be considered. Corrosion of reinforcement due to chloride attack or carbonating lead to decrease of the bearing capacity of reinforced concrete structures. This interaction of individual components represents a subsystem failure in the overall fault tree of bridge failure. The subsystem failure is called failure mechanism. These failure mechanisms are stored in PROBILAS as sample data sets. Once all failure modes and failure mechanisms are specified for the building, PROBILAS generates the fault tree for the complete structure (Klinzmann & Hosser 2005).

Among other things the following points where taken into account to ensure the bearing capacity of the bridge. On the one hand the shear failure at the support and on the other hand the flexure failure at midspan is to be analysed first. The failure of the transition joint, the penetration of de-icing salt linked with the consequence of a chloride-induced corrosion of the tendon anchorage assembly is a possible failure mechanism at the support. The prestressing is useful for the shear capacity at the support and can not be considered if the tendon anchorage assembly fails. For this reason the shear failure risk increases. If the transition joint does not fail,

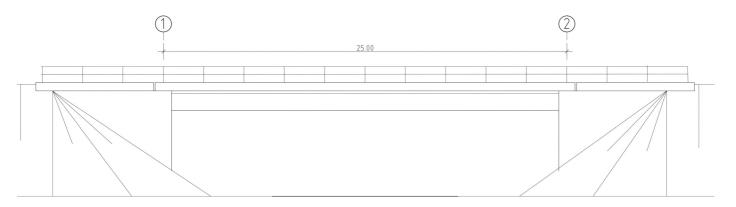


Figure 1. Bridge example, elevation



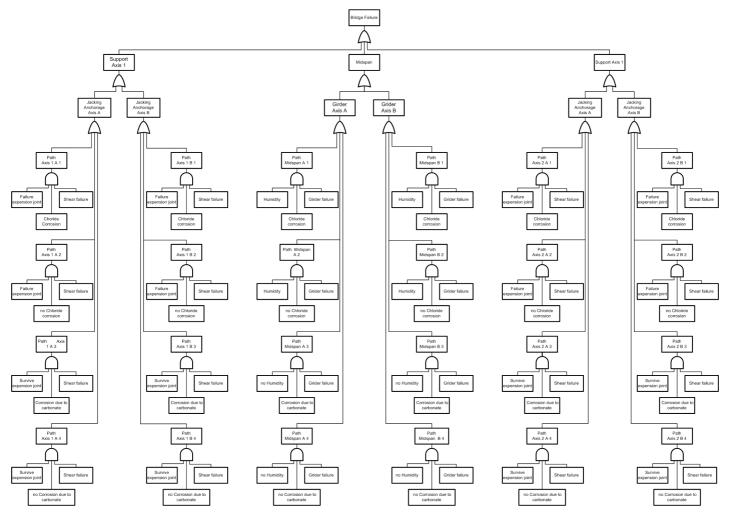


Figure 3. Fault tree

there is no chloride-induced corrosion due to missing of de-icing salt. For this case a corrosion is considered due to the carbonating of the concrete. In midspan the structure can fail by exceeding the flexure capacity or by fatigue fracture of the prestressing steel.

A corrosive degradation of the reinforcement here also leads to a reduction of the bearing capacity. Therefore either chloride-induced corrosion of the reinforcement or corrosion due to carbonating are taken into account in the failure mechanisms for midspan. The chemical and/or physical processes in the concrete do not exclude the simultaneous occurrence of chloride-inducing and carbonating. The transport of chloride in the concrete requires a moisture penetration of the concrete, which restrains again the carbonating process. Thus the independent view of both procedures in the reliability-based system assessment is a meaningful simplification.

For the structural health monitoring with special consideration of the bearing capacity the resulting fault tree is represented in Figure 3. Every component in this fault tree represents a limit-state. The modeling of the limit-states functions on this basis

for the shear failure at the support, the flexure failure and the fatigue failure in midspan will be described in this article.

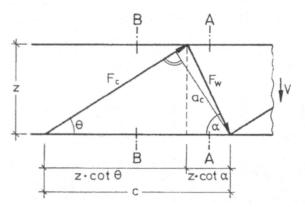


Figure 4: General truss model (Zilch & Rogge 2004)

#### 3.1.1 *Limit-state for shear failure*

A general truss model describes the shear properties (Fig. 4). The capacity of the ties is to be analysed with equation (1) and the capacity of the concrete compression struts with equation (2).



$$V_{R1} = A_{s,sti} * f_v * (\cot \theta + \cot \alpha) * \sin \alpha$$
 (1)

$$V_{R2} = \sigma_c * b_w * z * (\cot \theta + \cot \alpha) * \sin^2 \theta$$
 (2)

with  $A_{s,sti}$  = shear reinforcement per unit length,  $f_y$  = reinforcement yield strength, z = moment arm of the internal forces,  $\theta$  = angle of concrete compression struts,  $\alpha$  = angle of shear reinforcement,  $\sigma_c$  = compression concrete strength and  $b_w$  = girder thickness.

The shear action  $(V_s)$  due to dead load and life load stands against the shear resistance according to the equation (1) and (2) respectively. The prestressing decreases the shear action. At first, the reliability of the structure against shear failure is computed on the assumption of an intact anchor and with consideration of the pre-stressing.

The model uncertainties due to idealizations in the mechanical model and the load effect were taken into consideration by model uncertainty factors following (JCSS 2002).

The limit-state equations can be written in the general form Z = R - S. The component fails, when the resistance (R) is smaller than the action (S).

Table 1. Stochastic model for the shear failure

Basic Vari-	Sym-	Unit	Distribu-	Mean	Stan-
able	bol		tion		dard
					devia-
					tion
Compression	$f_c$	MN/m²	log-	38	2.28
concrete			normal		
strength					
Yield	$f_y$	MN/m³	log-	560	30
strength			normal		
Girder depth	h	m	constant	1.40	
Concrete	nom c	m	Beta	0.05	0.009
cover					
Shear rein-	$A_{s,sti}$	m²/m	constant	0.015	
forcement					
Angle of	α	0	constant	90	
shear rein-					
forcement					
Angel of con-	θ	0	rectan-	45	8.66
crete com-			gular		
pression					
struts					
Length	1	m	constant	25	
Dead load	g	MN/m	GN	0.098	0.00275
Live load	TLKW	MN	GN	0.406	0.065
Pre-stressing	P	MN	GN	15	0.3
force					
Tendon gra-	χ	-	constant	0.096	
dient					
Girder thick-	$b_{\rm w}$	m	constant	1.2	
ness					
Uncertainty	$\theta_{R}$	-	log-	1.0	0.1
of resistance			normal		
Uncertainty	$\theta_{\mathrm{S}}$	-	log-	1.0	0.1
of load effect	_		normal		

In this case the limit-state equation for shear failure results in:

$$Z_{1} = (A_{s,sti} * f_{v} * (\cot \theta + \cot \alpha) * \sin \alpha) * \theta_{R} - V_{S} * \theta_{S} (3)$$

$$Z_2 = (\sigma_c * b_w * z * (\cot \theta + \cot \alpha) * \sin^2 \theta) * \theta_R - V_S * \theta_S (4)$$

The stochastic models of the parameter are listed in Table 1.

#### 3.1.2 *Limit-state for flexure failure*

Another component in the fault tree illustrated in this chapter is the flexure capacity of the bridge. The flexure capacity for the bridge section can be calculated by the equilibrium of internal forces (Fig 5).

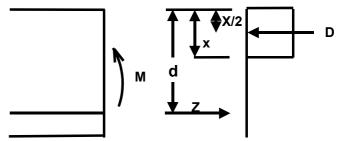


Figure 5. Internal force in case of bending moment

The associated equation for the flexure capacity can be described as follows.

$$M_{R} = A_{s} * f_{y} * d - \frac{1}{2} * A_{s} * f_{y} * \frac{A_{s} * f_{y}}{b * f_{c}}$$
 (5)

The resisting moment according to equation (5) will be compared with the acting moment due to dead load and life load.

A design vehicle (TLKW) with a middle weight of 400 kN as single load in midspan is simply assumed for the life load. The action–effect is calculated in equation (6).

$$M_{s} = \frac{g*l^{2}}{8} + \frac{TLKW*l}{4}$$
 (6)

This results in the limit state equation for flexure failure:

$$Z = \left(A_{s} * f_{y} * d - \frac{1}{2} * A_{s} * f_{y} * \frac{A_{s} * f_{y}}{b * f_{c}}\right) * \theta_{R}$$

$$-\left(\frac{g * l^{2}}{8} + \frac{TLKW * l}{4}\right) * \theta_{s}$$
(7)

The stochastic model for the related parameter in equation 7 are listed in Table 2. In the reliability analysing also a model uncertainty factor is kept in mind.



Table 2. Stochastic model for the flexure failure

Basic Vari-	Sym-	Unit	Distribu-	Mean	Stan-
able	bol		tion		dard
					devia-
					tion
Compression	$f_c$	MN/m <sup>2</sup>	log-	28	2.28
concrete			normal		
strength					
Yield	$f_y$	MN/m³	log-	1610	30
strength			normal		
Reinforce-	$A_{s}$	m <sup>2</sup>	constant	0.0134	
ment					
Girder depth	h	m	constant	1.40	
Effective	b	m	constant	5.00	
width					
Moment arm	d	d = h-a			
Length	1	m	constant	25	
Dead load	g	MN/m	GN	0.098	0.00275
Live load	TLKW	MN	GN	0.400	0.065
Uncertainty	$\theta_{R}$	-	log-	1.2	0.15
of resistance			normal		
Uncertainty	$\theta_{\mathrm{S}}$	-	log-	1.0	0.1
of load effect			normal		

Furthermore the limit-state for the flexure failure also includes a corrosive degradation of the reinforcement. Due to penetrating chloride or the carbonating of the concrete corrosion of the reinforcement can occur. A probabilistic description of the processes in concrete are available in the literature (Gehlen 2000). An analytical description of the corrosion progress of the reinforcement is pretty difficult up to now.

The calculation module of PROBILAS makes a prognosis of the reliability. Therefore a time-step procedure is used. In this limit-state the linear decrease of the reinforcement cross section is assumed to 10 % during 10 years after initial corrosion.

The result is a trend for the reduction of the reliability with respect to flexure failure. Newly measured values for the corrosive degradation adapt the prognosis to the current state.

#### 3.1.3 *Limit-state for fatigue failure*

Apart from the bending capacity also the limit-state for fatigue failure of the tendons in midspan is to be analyzed.

The describing model for fatigue failure is based on the Palmgren-Miner-hypothesis. This hypothesis means that alternating loading of the construction causes damages of the materials, which adds themselves until a critical damage value is reached. With the exceeding of this value fracture occurs. The damage progress of the tendon is described by the dimensionless damage factor D.

$$D = \sum_{i} \frac{n(\Delta \sigma_{i})}{N(\Delta \sigma_{i})}$$
 (8)

In the equation is  $n(\Delta\sigma_i)$  the number of load cycle with the stress difference  $\Delta\sigma_i$ .  $N(\Delta\sigma_i)$  is the maximum number of alternations for the stress difference  $\Delta\sigma_i$ . This maximum number can be read off directly

from the S-N curve. Per definition the fatigue failure occurs when the dimensionless damage factor D reaches the value 1. On basis of the Palmgren-Miner-hypothesis one can define a limit-state equation for the fatigue failure (Buba 2004).

$$Z = D_R - D_S \tag{9}$$

In the context of the first building assessment the number of  $0.5*10^6$  trucks is assumed as the annual volume of traffic. The annual volume of traffic is associated with the fatigue load model 4 (FLM) from Eurocode 1 (EN 1991-3 2003).

Based on a sharpening of the stochastic model for the traffic load as a result of structural health monitoring a re-evaluation of the structure is possible

The chosen load model includes five load steps which are substitutional for a damage-equivalent volume of traffic. For each load step the stress difference results from equation (10).

$$\Delta \sigma_i = f_{M-\sigma}(\text{max. M}) - f_{M-\sigma}(\text{min. M})$$
 (10)

The function  $f_{M-\sigma}$  is the relationship between the bending moment and the stress in the tendon. The bridge is pre-stressed with 8 tendons per girder (Fig. 6). The post tension system using strands Dywidag AS-140mm<sup>2</sup> is installed. Figure 6: General truss model (Zilch & Rogge 2004)

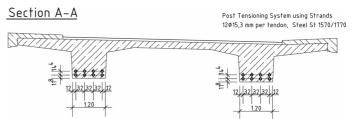


Figure 7. Bridge section

In the limit-state for fatigue failure only the prestressed tendons are considerd. Therefore the relationship  $(f_{M-\sigma})$  in Fig 7 between bending moment and the stress in the tendon for bridge section (Fig 6) results.

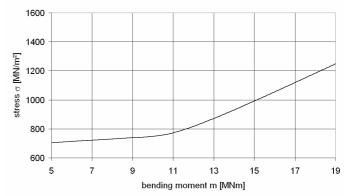


Figure 8: Moment stress curve  $(f_{M-\sigma})$ 

With the stress differences determined in this way the maximum number of alternations can be read the S-N curve.



The used S-N curve is leaned against the representation in the Eurocode 2 (EN 1992-1-1 2004). For curved tendons in steel ducts the function for the maximum number of alternations is N =  ${\rm e}^{28,179}$  \*  $\Delta\sigma^{-3}$  for a stress difference of  $\Delta\sigma > 120$  MN/m² and N =  ${\rm e}^{47,323}$  \*  $\Delta\sigma^{-7}$  for a stress difference of  $\Delta\sigma < 120$  MN/m². The values are assumed as 90-percentile. Further the coefficient of variation for the maximum number of alternations is assumed as 30 %.

In the analysis for the fatigue failure the same damage function as in the analysis for the flexure failure, i.e. a cross section decrease of 10% during 10 years, is used to prognose the reliability. This is considered both for the relationship between bending moment and stress in the tendon and in the S-N curve.

#### 4 RESULTS

# 4.1 Limit-state for shear failure

The reliability of the component shear failure is analysed with and without consideration of the prestressing. The analysis was independent from other components in the fault tree for the assessment shown here.

The results of the FORM/SORM reliability analysis are shown in Fig. 7. Thus it appears that the bridge is very durable in the support range. The safety index  $\beta$  is sufficient in the limit-state for the failure of both the ties and the concrete compression struts.

After a failure of the tendon anchorage assembly the beneficial effect for the shear capacity by the pre-stressing can't be considered.

The FORM/SORM reliability analysis for this case shows that the structure has still a large safety margin concerning shear failure at the support.

# 4.2 Limit-state for flexure failure

The FORM/SORM reliability analysis for the flexure failure was accomplished on the assumption of a corrosive degradation of the reinforcement.

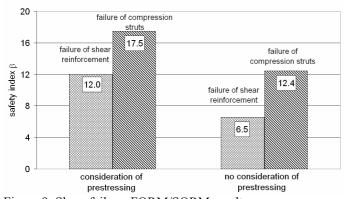


Figure 9: Shear failure, FORM/SORM results

Under these conditions the results explained in Fig. 8 can be obtained. The figure shows the trend for the safety index  $\beta$  and the probability of failure  $p_f$  ( $p_f$ = $\Phi$ (- $\beta$ )) for 10 years after the initial corrosion. The safety index  $\beta$  (probability of failure  $p_f$ ) is perfectly sufficient at the beginning and decreases hardly in the regarded time frame. With these results a sudden failure by exceeding the flexure capacity is not expected.

### 4.3 Limit state for fatigue failure

According to the view of the safety level for the flexure capacity during a defined duration also the safety level against fatigue failure was prognosticated.

The safety index  $\beta$  and the probability of failure  $p_f$  ( $p_f$ = $\Phi$ (- $\beta$ )) are shown in Fig 9 for a time of 10 years.

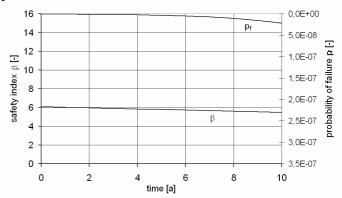


Figure 10: Flexure failure, FORM/SORM results

By increasing the load cycles the damage factor D grows and consequently the safety index  $\beta$  decreases. Without consideration of a damage or additional actions for the tendon the value of the safety index  $\beta$  and respectively the probability of failure is acceptable not only for the time of 10 years but also for 100 years.

With consideration of the represented damage it comes to a reduction of the safety level, which leads to the fact that after 10 years rehabilitation measures are needed. Due to the strong reduction of the safety level it appears meaningfully to start with preventive rehabilitation measures earlier than after 10 years.

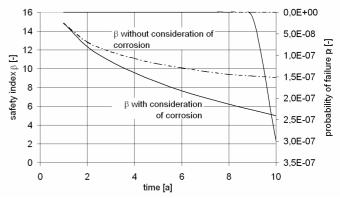


Figure 11: Fatigue failure, FORM/SORM results



Beside the consideration of a damage for the prognosis of the safety level of individual components, a building evaluation with continuously measured values from the structural health monitoring is necessary, in order to make further decisions concerning the monitoring and maintenance.

# 4.4 Evaluation of the results

On the basis of different components in the system bridge, the modeling and reliability analysis of limit-states was shown. Due to these few results consequences for the further structural health monitoring already become apparent.

After an assumed failure of the tendon anchorage assembly the safety index  $\beta$  for shear failure decreases, however an acute danger does not exist.

The decrease of the safety level for flexure failure was very small, so that a sudden failure does not have to be assumed. Here it is meaningful to use decision making aids to define measuring intervals, in order to compare the prognosis with the current building condition

With the limit-state for fatigue failure the comparison between the prognosis and the current building condition is necessary, since the safety index  $\beta$  drops very strongly.

#### 5 CONCLUSION

In this contribution the application of the building inspection and assessment system PROBILAS using a bridge as an example was shown. Especially the durable bearing capacity is regarded.

First the probabilistic model of the building in form of a fault tree was generated and subsequently the modeling of the limit-states was shown at selected components.

For the set up of the limit-state equations and the specification of the stochastic model for the parameters the system PROBILAS will reproach sample data sets. These example data sets can be adapted user-specifically. Beside the specification of the limit-state equations the probability of failure, e.g. for the failure of the tendon anchorage assembly, can be indicated directly as numerical value.

The independent assessment of individual components was possible under the precondition that the individual components are not correlated.

From the results the weak points can be identified, which have to be monitored with priority. Due to the prognosis represented here measuring intervals can be specified using decision making aids which will be developed. These decision making aids are Part of PROBILAS just like sample data sets and will help to optimize the structural health monitoring.

#### 6 ACKNOWLEDGMENTS

The research presented in this paper is supported by the "Deutsche Forschungsgemeinschaft" within the Collaborative Research Center "SFB 477". The authors would like to express their appreciation for the financial support.

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