Game Theory and Real Options:
An alternative to the replicating portfolio

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1. Motivation
2. Case Study
   - NPV valuation
   - Real Option valuation and related problems
3. Competition and Game Theory
4. Some results

The problem

- This paper contributes to a novel literature which joins Real Option Theory and Game Theory
- Literature on Real Estate:
  - Only RO: Titman (1985), Williams (1993), Grenadier (1995) and many others
  - RO-GT: Smit and Ankum (1993), Grenadier (1996) and few others
  - Constant BIG PROBLEM: short sales/replicable portfolio
- We focus on Multiple optimal investment decisions
- Offer a first solution to the big problem

Case Study

- The site was acquired at the price of £12.78m
- The difference between the annual cost of £150k to keep the strategic option open, and the annual income generated by a car park managed on the site is marginal
  - We assume that there is no either cost or income in deferment other than financial costs related to discounting (i.e. the dividend is equal to zero)
- The local authority wishes to see the site completely developed and has already granted planning permissions for the actual development to be started within the next 5 years. Whenever the investor wishes to abandon the scheme within the next 5 years, she has to sell it back to the local authority at a fixed price of £8m

NPV valuation

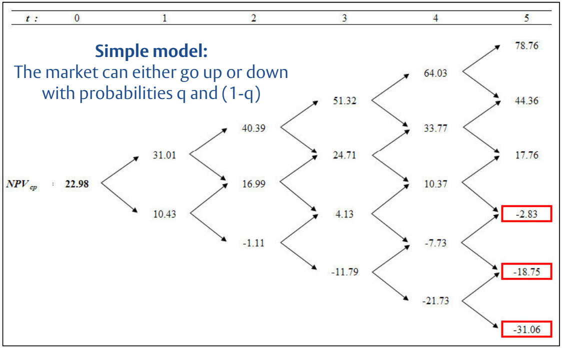
Month	0	3	6	9	12	15	18
Time Index (t)	0	1	2	3	4	5	6
Property Sale	-	-	-	-	-	-	-
Land Acquisition	-12.78	-	-	-	-	-	-
Construction Costs	-0.59	-1.79	-2.00	-2.07	-2.89	-5.09	-6.06
Site Enabling	-0.13	-0.05	-	-	-	-	-
Prof Fees	-0.34	-0.22	-0.24	-0.25	-0.35	-0.61	-0.73
Other Fees	-0.51	-	-	-	-0.05	-	-
FCF_t	-14.34	-2.05	-2.24	-2.32	-3.29	-5.71	-6.79

Month	21	24	27	30	33	36	39
Time Index (t)	7	8	9	10	11	12	13
Property Sale	-	-	-	-	-	-	105.76
Land Acquisition	-	-	-	-	-	-	-
Construction Costs	-8.44	-10.06	-7.21	-5.75	-3.80	-1.10	-
Site Enabling	-	-	-	-	-	-	-
Prof Fees	-1.01	-1.21	-0.87	-0.69	-0.46	-0.13	-
Other Fees	-	-0.37	-	-	-	-0.32	-0.84
FCF_t	-9.45	-11.63	-8.08	-6.44	-4.25	-1.55	104.92

Annual WACC (k)	9.00%
Quaterly WACC (k_Q)	2.18%

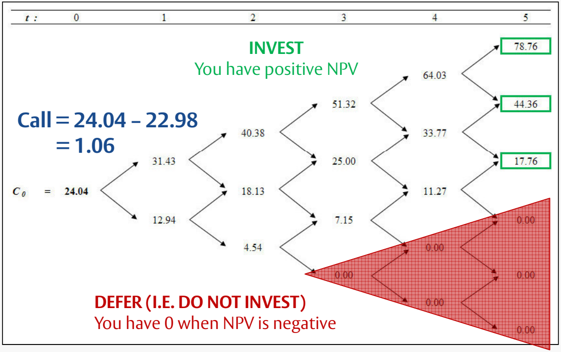
NPV_p	10.20
NPV_{cp}	22.98
PV (selling price)	79.93
PV (construction phaze)	56.95

NPV reconstruction



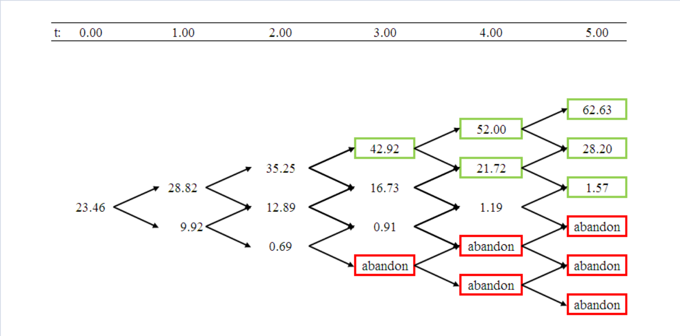
- Upward jump $\equiv u = \exp^{\sigma\sqrt{\Delta t}}$
- Downward jump $\equiv d = \exp^{-\sigma\sqrt{\Delta t}}$

Deferral Option Value (incl. NPV)



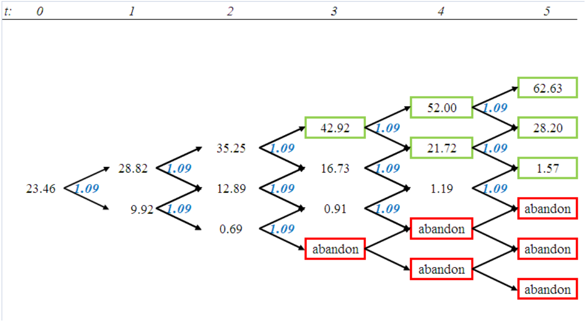
- Option value
  $\equiv C_t = \exp^{-r\Delta t}(q \max[V_u, C_{t+\Delta t,u}] + (1-q)\max[V_d, C_{t+\Delta t,d}])$
- EMM $q = \frac{\exp^{r_F * \Delta t} - d}{\dots}$; Which replicating portfolio??

Decision Tree Analysis



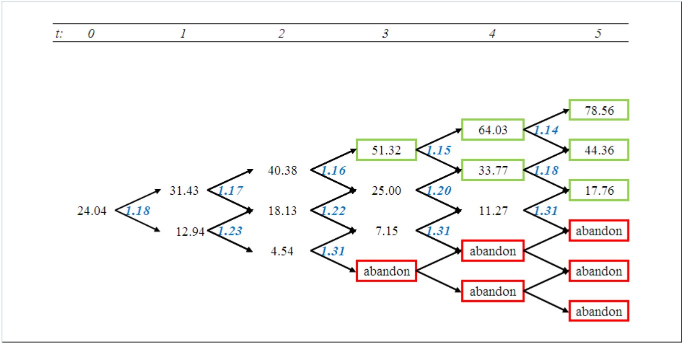
- Proposed solution by Boris, *JACF 2005*
- EMM $q = \frac{\exp^{rW*\Delta t} - d}{u - d}$

A problem of arbitrage (DTA)



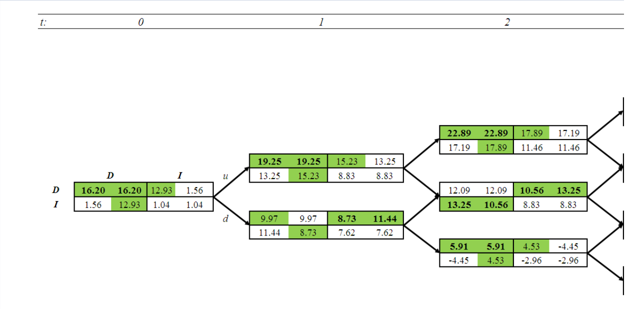
- Problem: Risk-adjusted discount rates are constant
- Arbitrage: Is not risk changing along the tree??

A problem of arbitrage (*back to ROA*)



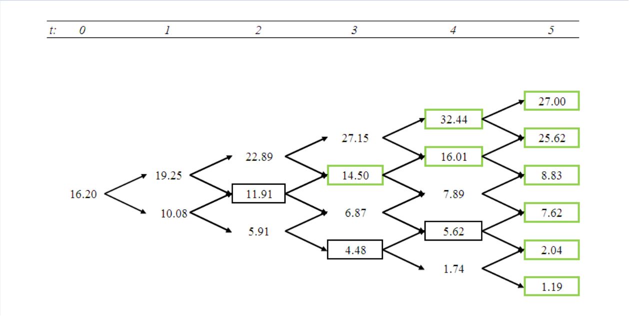
- Risk-adjusted discount rates are **<u>not</u>** constant
- No Arbitrage given replicable portfolio

Real Option and Game Theory Analysis



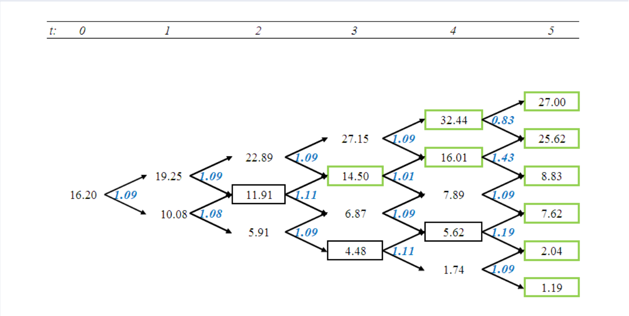
- Cournot Model: $P = a - b\overline{Q}$, $Q_L > Q_S > Q_F$
- Two options: (i) Defer and (ii) Decide the Size
- No replicable portfolio assumed, EMM $q = \dfrac{\exp^{rW*\Delta t} - d}{u - d}$

Obtained Decision Tree and Valuation



- Which *number* when multiple equilibria? Gabrieli and Marcato, 2010
- No replicable portfolio assumed, what about arbitrage?

Obtained Decision Tree and Valuation (2)



- Risk-adjusted discount factor varies
- Arbitrage opportunities **not** based on replicable portfolio have been excluded

The impact of competition

	b=0.1	b=0.3	b=0.5
DEFER/NPV	20 %	24 %	10 %

The impact of equilibrium selection rules

	Optimistic	Average	Pessimistic
DEFER/NPV	11.5 %	9.2 %	8.56 %

Conclusion

- Contribution
    - Comparison of various approaches
    - Risk-varying discount rates
    - No evident arbitrage opportunities
- Questions ? Suggestions ?