

## A density function of the appraisal value

Calculation and evaluation of the empirical density function of the appraisal value based on comparison method, spatial correlation techniques, resampling methods, compliant with the Spanish legal framework

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### ABSTRACT

**Purpose** - The objective of this paper is the presentation of a methodology for calculating and evaluating the empiric probability function of the market value for a defined property. As it is commonly known 'The value of an object is not unique' (Roca, 1986), but is defined by its distribution function.

**Design/methodology/approach** – The tools necessary to develop this project were: the expert methods theory, accurate information in the area of construction techniques, statistical resampling methods, spatial correlation and regression methods, the study of the empirical distribution functions and the statistics which define the distribution functions, GIS tools, the hedonic functions, simulation methods, discriminant methods and the studies of legal procedures related to the valuation studies process.

**Findings** - Thanks to nowadays computation methods it has been possible to achieve a representation of the market value of a property by taking into consideration the information on the set of samples.

The empirical distribution is not only the image of the behavior of the value; it is an indicator of the quality and the effectiveness of the solution and an indicator of the complexity of the evaluated zone.

Based on this concept, it is possible to decide based on the statistical definition of the distribution curve, how and where the solution obtained could be employed.

The statistical method of analysis of spatial correlation can be of great utility in the property valuation world, but a previous technical procedure is necessary to guarantee the quality of data.

**Practical implications** - The hybrid AVM presented in this paper can be a tool of enormous help, not only for the valuation entities in their individual valuation procedures, but also for institutions that aim to perform mass valuations.

**Originality /value** - It is a high quality solution that can provide substantial assistance to those officials responsible for conducting the mass valuation processes required by BASEL II and III. It can also be employed as an auditing tool for legal frameworks like BASEL II and III and also for the everyday valuation processes performed within a company when the results of the valuation arise from different providers.

**Keywords** - Real estate, mortgage, BASEL, AVM, appraisal, valuation, kriging, bootstrap, density function.

**Paper Type** - Research paper.

## 1. Introduction

The automatic appraisal mechanisms known as AVMs (automated valuation models) ought to be the tool used by all agents involved in the mortgage valuation process to value and control the total portfolio of mortgages in accordance to the international agreements, namely BASEL II and BASEL III.

The appearance of AVMs in Spain dates back to the beginning of the year 2000. Those early models aimed to provide the value of a property under any given circumstances, and consisted of more or less complex calculators that enabled a swift processing of calculations after receiving all relevant data.

In 2003, CATSA (a Spanish valuation company) signed a cooperation agreement with the UPC, aiming to improve the existing methods. In 2004 their first results were published (Garcia-Almirall, P. et al, 2004).

Beginning of 2005, in the light of the results obtained which showed that the quality of the methods developed wouldn't allow to replace a human appraiser, in spite of the excellent theoretical results ( $R^2 > 0.89$ ), it was decided by the participants and promoters of the project to reorient the project and define an Expert System (E.S.) aimed to generate the appraised value by means of an automated comparison method and would furthermore comply with the Spanish Property Appraisal Order ECO805/2003. The decision to employ the comparison method arose from the fact that this method is employed in 95% of all mortgage valuation procedures.

The new system had to be able to: study the market of comparable samples, select the most significant ones, homogenize the selected samples and finally determine the market value based on the comparison method and the existing legal framework.

The goal was to achieve that the AVM was not distinguishable from the manual process carried out by a professional. In addition to providing high quality solutions, it had to be able to identify those situations in which, based on the available data, a solution can't be provided by the system with the sufficient quality required.

This situation would imply the involvement of a technician in the process, converting the system into a hybrid AVM. It is precisely at this point when the second objective of the project was defined, namely its need for continuous improvement in order to minimize the participation of technical experts in the process.

Based on prior studies by the technical team of CATSA, a list of conditions under which the AVM would not be able to provide an automatic solution and thus the intervention of a qualified technician would be required, was established as follows:

- The information available was not sufficient to run the calculations.
- The outcome of the process was inconsistent, due to discrepancies among the available samples.

A testing period of two years ended up defining a system based on a new mechanism for entering data that enabled the system to use the totality of the information entered as qualitative or quantitative variables. New rules, heuristics, procedures and models were designed to allow a correct selection of comparable samples and a subsequent homogenization of the samples according to the valuation object (Section 2).

Section 3 presents a summary of all the statistical tools used in the definition of the AVM as for example the spatial statistical methods, the bootstrap methods or the cross-validation methods. All of them can be consulted in Barris, J. (2008). In addition, it presents a brief description of the discrimination methods used by the Expert System in order to decide if the solution calculated by the system is acceptable or not (Section 4).

Section 5 illustrates the calculation algorithm by showing different results related to the method of acceptance of the solution on the basis of the shape of the empirical density function obtained. Additionally, there is a summary of results attached.

Finally, section 6 presents the conclusions.

## **2. The selection and homogenization of comparable samples**

It can be assumed that two objects are comparable when they belong to the same market, which means that both objects are possible options for the same buyer (Roca Cladera, 1986).

On the basis of the above definition we can establish four sets of restrictions with the objective of defining the concept of comparability:

- Restrictions due to the age of the sample
- Market restrictions
- Location restrictions and 'shadow' restrictions
- Restrictions arising from the difference in value caused by homogenizations.

The samples should be monitored periodically with regards to market fluctuations. Therefore, prior to initiating the calculation process it is necessary to eliminate irrelevant samples according to the parameters established for the market of the valuation in process.

### **2.1. The restriction due to the age of the sample**

It is necessary to differentiate between the primary market (objects less than 2 years old) and the secondary market (objects more than 2 years old).

The age restriction therefore establishes the use of those samples only that belong to the same market (primary or secondary) as the object to be valued.

### **2.2. Market restrictions**

The goal of each market restrictions is to determine whether the object is comparable or not. The main market restrictions are:

- Typology: Comparable samples are those samples belonging to the same typology
- Surface: The comparability is defined by a mathematical function (Figure 2.1)
- The height, defined by floors, when there is no elevator
- The difference in number of floors between the samples and the object when there is no elevator
- The number of bedrooms and bathrooms

As an example of this type of restrictions the figure below shows the restriction of comparability based on the surface of the object to be valued. This figure was obtained from internal studies conducted by CATSA. Its contrastation was performed based on a survey of 30 appraisers of the company.

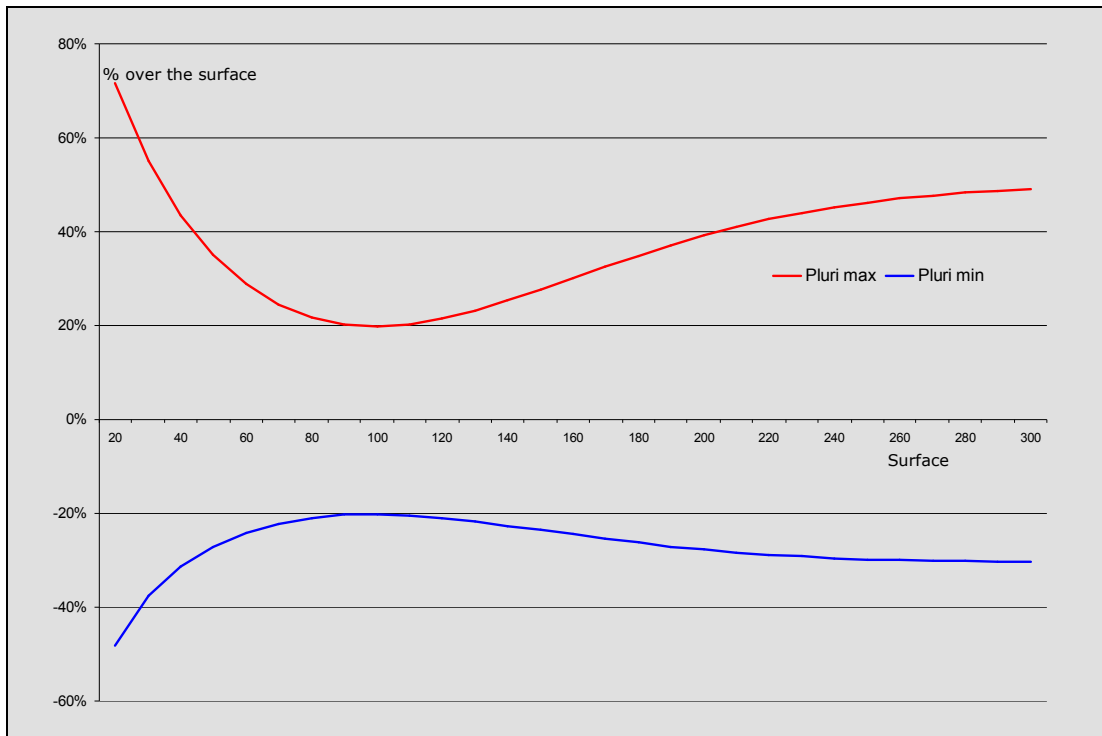


Figure 2.1.: The construction type: Example of comparability based on the surface in multi-family houses

### 2.3. Location restrictions and 'shadow' restrictions

The proximity of samples has been a mainstay of the AVM and the hybrid AVM models, but the discontinuity comparisons have always been used by appraisers in order to overcome the problems arising from the lack of comparable samples in a given area.

The kriging models for comparable samples are able to use this information correctly, in hand with a correct entering and use of variables such as quality of views, type of street and area, proximity to certain urban elements that can be beneficial or not or any other feature connected with the social opportunity cost, and those variables mentioned in previous sections, so that there will be no need to establish any initial distance constraints.

### 2.4. The restrictions arising from the difference in value caused by homogenizations

The homogenization processes play an important role in finding logical comparable samples. It is known that, in general, regressive methods, heuristics and mathematical formulations are able to perform any homogenization on a mathematical level. However, the underlying idea should be to be able to compare properties that could be consumed by a single buyer and thus the methodology should go in that direction.

The features used in the homogenization process are the following: the surface, number of bathrooms, number of rooms, age of the construction, the actual state, the quality of its finish, the plant and the availability or not of an elevator. Based on these characteristics, a number of functions and heuristics have been established with the aim of finding the differentials of value that causes the differences

between the valuation object and the sample, and of establishing the value of the characteristic of the object as pivotal value of the function or heuristic.

For each of these characteristics, a homogenization function has been established, which includes construction costs, quality and percentage of cost of its finish without equipment.

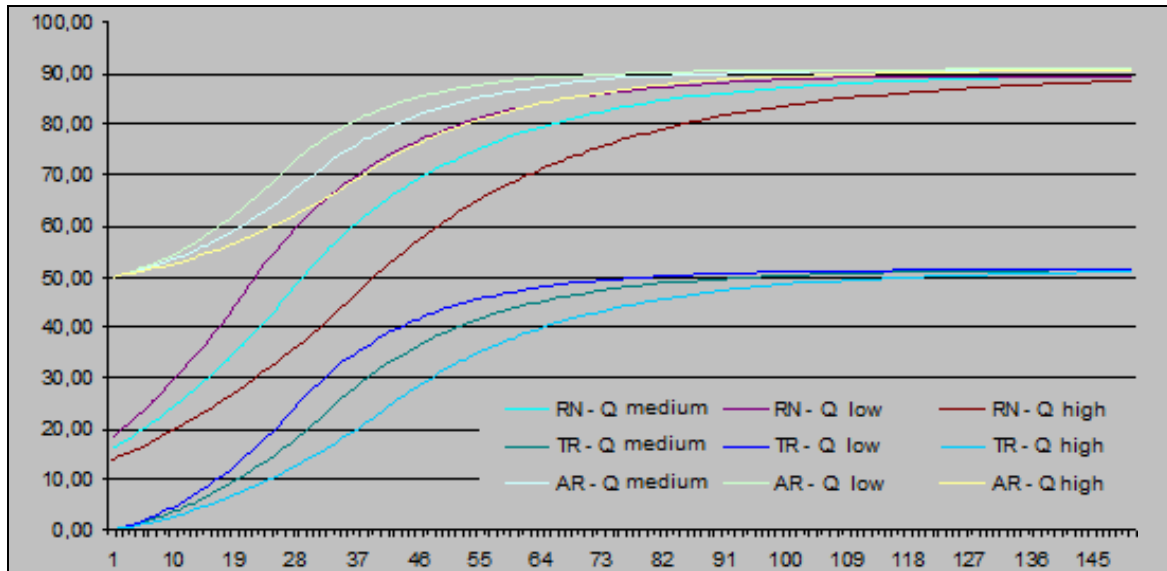


Figure 2.2.: ARC curves based on the state and quality.

Figure 2.2. illustrates the depreciation functions of the qualities (high, medium and low) according to their antiquity and definition of state (which are: TR - totally renovated, AR – advisable to reform and RN – reforms are necessary) of the valuated object.

The results of this process are the six differentials in monetary value presented in Table 2.1.

The conversion methods to euro of the differences between the object and the sample are based on studies conducted by the technical staff of CATSA.

Monetary difference associated to the surface difference	$D_{surf}$	$Abs(D_{surf})$
Monetary difference associated to the difference number of baths	$D_{bath}$	$Abs(D_{bath})$
Monetary difference associated to the difference number of rooms	$D_{room}$	$Abs(D_{room})$
Monetary difference associated to the ARC difference	$D_{arc}$	$Abs(D_{arc})$
Monetary difference associated to the quality difference	$D_{qual}$	$Abs(D_{qual})$
Monetary difference associated to the elevator indications	$D_{lift}$	$Abs(D_{lift})$

Table 2.1. Differentials between characteristics in monetary value.

The homogenized value of each of the samples is obtained as follows:

$$VH_i = PM_i + D_i = PM_i + (D_{i,surf} + D_{i,bath} + D_{i,room} + D_{i,arc} + D_{i,qual} + D_{i,lift}) \quad (1)$$

where  $VH_i$  is the homogenized value of the  $i$ -th sample,  $PM_i$  is the market price,  $D_i$  is the homogenization required in terms of money to acquire 'identical' properties in terms of physical characteristics, state and quality.

The quality verification of the homogenization methodology was performed by means of an exercise conducted by 21 technicians in which each technician had to execute 120 homogenizations. The total number of comparisons was 2.520.

The screenshot shows a software interface for data collection and comparison. At the top, it displays 'Objeto' with details: 80m², 3 hb, 1 bñ, 3º, Asc, 1950, and 'Algunas reformas necesarias'. A 'Valor medio del Objeto = 250.000 euros' is shown. Below this is a table with 6 samples (Muestra 1 to Muestra 6) with varying areas and similar characteristics. At the bottom, there are comparison options: 'Comparable?' with radio buttons for 'No comparable' and 'Comparable', and 'Homogeneización' with dropdown menus for 'La muestra es' and input fields for 'Ajuste en euros' (Máximo and Mínimo).

Figure 2.3: Data collection and comparison software used

Based on the samples considered as comparable by the Expert System a means  $t$ -test was performed between 'the average price per square meter defined by the technicians' and 'the average price per square meter defined by the expert system'.

$$H_0 : \mu_{\text{technician s}} = \mu_{E.S.} \quad (2)$$

$$H_1 : \mu_{\text{technician s}} \neq \mu_{E.S.}$$

The null hypothesis could not be rejected, and therefore it was assumed that there were no statistically significant differences between the procedure of the technical team and that of the E.S.

The correlation coefficient between the average value of the technicians and the value of the E.S. was 0.89.

Furthermore, two rules for the non-comparability were defined based on the value differentials of the homogenization process.

$IF \left( Abs(D_{i,surf}) + Abs(D_{i,bath}) + Abs(D_{i,room}) + Abs(D_{i,arc}) + Abs(D_{i,lift}) + Abs(VH_i) \right) OR$   
 $IF Abs \left( \frac{VH_i - PM_i}{PM_i} \right) > 0.17 THEN the sample is not comparable$

Table 2.2: Rule of acceptable homogenization.

### 3. Theoretical tools used in the process of calculating the appraised value

#### 3.1. Spatial estimation methods

Cressie (1991) states that for those data sets that are associated with a spatial component it should be assumed that the similarities between nearby data are more evident than among those data that are farther apart. It is evident that the values of real estate objects have always been attached to an important spatial component, which is why it has always been important to exploit and consider estimation methods that are based on spatial components.

In order to compare the quality of the results, different estimation methods were employed. The simplest estimation method is based on reasoning that all samples accepted as comparable samples have to participate with the same weight in the calculation of the appraised value. This method has been named as 'method of identical weights'.

Three spatial interpolation methods were employed, namely inverse distance, moving averages and radial functions, which based its calculation of the weights of each of the samples on the distance between the object and the samples.

Finally, two spatial estimation methods were proposed - isotropic and anisotropic kriging (Krige, D.G. 1951) - which introduce a spatial correlation matrix that defines the end of the catchment area of the samples and in the case of anisotropic kriging the maximum influence direction.

#### 3.2. Bootstrap methods. The permutations method.

The techniques known as resampling bootstrap methods (Bradley E. 1979, Tibshirani 1993) and as permutations method have enabled to represent the density function of the appraised value and to eliminate the influence and bias that could generate misplaced samples, that might have been entered incorrectly or had simply been entered with characteristics of localization that were differentiated but not distinguishable within the sample space, and due to their proximity have gained important weight in any of the estimation methods presented.

#### 3.3. The cross-validation methods. The one-to-leave method.

The cross-validation method (Devijver, P.A., 1982) and specifically the method of one-to-leave should facilitate a joint evaluation of the differences between the actual and the estimated values, allowing to establish the quality of the estimates and the homogeneity between samples.



These mechanisms can be used during the sample generation process. The system performs an evaluation using the same mechanisms as the ones used for conducting the actual valuation in order to determine if the samples are homogeneous within the area and the Expert System generates the required notifications for its control.

This quality control mechanism of samples and their manual review are very useful and they are actually the basis for the proper functioning of the AVMs.

### 3.4. The discriminant methods

Different methods of discriminant analysis have been employed to differentiate the empirical distribution functions that reach acceptable solutions from those that don't reach them. We will use as variables of the discriminant analysis all those statistics and measures that provide empirical distribution functions.

To model the binary variable "There are several urban areas/There is a single uniform space", a logistic regression will be employed.

## 4. Mechanisms to evaluate solutions

### 4.1. The empirical density function of the appraised value and its validity as a solution

It is well known that the value of a property is not unique (Roca, 1986), but has a distribution function. Thanks to the existing computation methods it is nowadays possible to achieve a representation of the property value based on the set of the sample information available.

#### 4.1.1. Calculation of the empirical density function of the valuation

The procedure used to produce the density function is the following:

- All possible subsets of 6 samples without repetition (bootstrap resample) are generated based on the initial set of comparable and homogenized samples that have passed all previous filter.
- From each resample it calculates the assessed value for each of the methods presented.
- This process will get  $C_6^m$  (where m is the number of comparable samples) solutions that allow to represent the empirical density function of the appraised value.

The bootstrap procedure provides all the necessary statistics for calculating the confidence intervals and the hypothesis tests.

We will chose as appraised value the midpoint value of the modal interval of the empirical density function and from all the resamples that equal the midpoint value, the one with the smaller EVC4 should be selected.

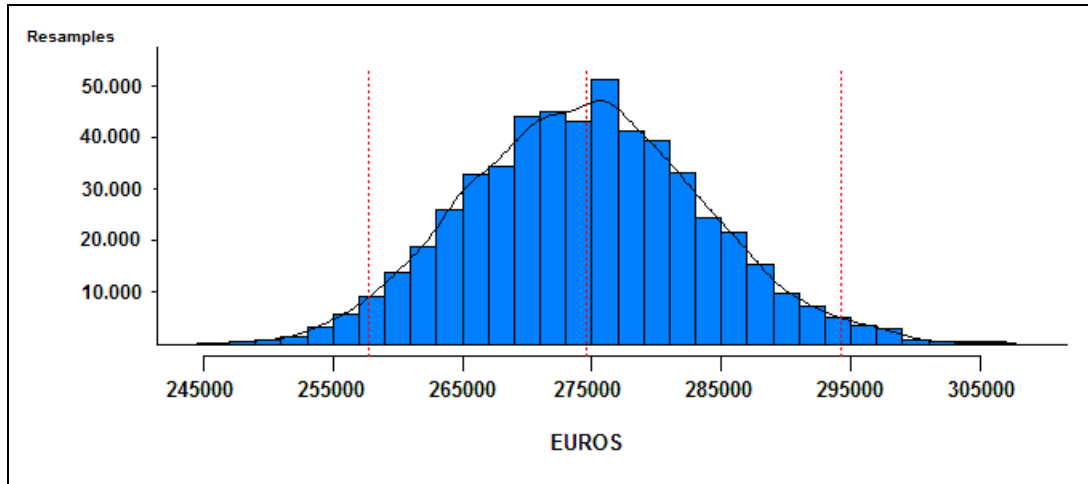


Figure 4.1.: Representation of distribution from the bootstrap estimations.

The  $EVC_4$  is defined as follows:

$$EVC_4 = \sum_{i=1}^N \text{abs} \left( \frac{Vh_i - Vhe_{-i}}{Vh_i} \right) \cdot p_i \quad (3)$$

where  $p_i$  is the weight of the sample in the valuation of the object.

#### 4.1.2. Theoretical approach and assumptions

From the shape of the density function it is possible to establish the validity of the results of the estimation. The logic of the process is as follows:

- If the area where the property to be valued is located is uniform, then the density function will have a single modal interval (local maximum).
- If the area where the object is located is an intersection of different urban areas that have different functions of value, then the density function is not unimodal (different local maxima) and the average value could not be considered a solution because it does not represent any of the areas.

The information available when performing an automatic assessment process is the following:

- The number of available samples,
- The market value of each of the samples;

and unknown information is:

- the distributive form of the area,
- if there are several urban sectors around the object to be valued,
- if there is more than one distribution function.

Therefore, it is necessary to perform an empirical representation of the density function, since the theoretical distribution is unknown. The problem is that a large number of samples are required in order to achieve an accurate representation of any density function.

The number of samples that usually are available in the field of real estate valuations, due to intrinsic characteristics of the market, hardly reaches the minimum amount needed to perform these representations with sufficient precision to study the modal behavior. In this sense, a good solution when performing the representations of the density function would be to generate bootstrap resamples. This procedure in turn might also lead to problems when identifying modal intervals.

To maintain a parallel with the ECO805/2003, each of the bootstrap resamples were obtained as a weighted average of six real samples. The total number of bootstrap samples was  $\binom{n}{6}$ .

#### 4.1.3. Logics and mechanisms for identifying valid empirical density functions

The use of bootstrap methods to calculate the assessed value produces less biased results than those obtained by statistics calculated directly on the sample. This means that if an abnormal value is entered in a sample (e.g. belonging to another population, with values very different from the study population), this value does not influence the final outcome of the bootstrap.

But what happens when the number of abnormal samples which belong to another population is high enough to alter the outcome of the bootstrap method? Is it possible to make a prediction when this situation occurs?

To resolve this issue a simulation experiment was performed. This experiment is the starting point for automated validation of solutions.

The characteristics of the data used in the simulation are:

- Only the observed data is known.
- It is unknown if they come from one or more populations.
- It is unknown what amount of data belongs o each of the subpopulations.
- It is unknown what are the associated distribution functions.

The simulation is defined as follows:

- Based on the real differences obtained by simulation between the two subsamples, the dichotomous variable needed for the analysis process has been defined.
- In the simulation process, samples from beta and normal density functions have been mixed, which increases the difficulty of the predictive model.
- The range of variation between the theoretical differences of the population means has been set between 0% and 20%.

- It was considered that differences beyond 13%, in average, between the two simulated populations should be indicated by the system as samples coming from different populations.
- In the simulation process, it was observed that the number of samples from 'different' populations equaled the number of samples considered not to be 'different'.
- In the process 26 logistic regressions were defined, one for each possible sample size. Sample sizes could range between 6 and 32 samples. However, the result obtained from 6 samples cannot generate bootstrap resamples.
- The acceptance algorithm is based on the results of logistic regressions and on a set of logical rules that improve the predictive results and are based on the available information.

If the samples come from 'different' populations then the algorithm should identify this situation and not resolve the valuation automatically. Whereas, if the samples come from populations that can not be considered 'different', then the result of the valuation method based on bootstrap methods would be considered as correct. The results of the process are shown in Table 4.1.

The error or disagreement between the reality and the prediction are given by two situations:

- There is a single population; however the test results indicate the existence of two or more populations. This error implies the need for manual validation to achieve a correct solution. This type of error has been named as 'error with manual validation'.
- There are two or more different sets of value, but the test does not distinguish them (serious error). The situations in which this error occurs imply the acceptance of the automatically appraised value. Consequently, this is precisely the type of error that must be minimized. This type of error has been named as 'uncontrolled error'.

After running 10 modeling processes, each of it with about 12000 simulations - half of which came from a single population and the other half were results generated from two different populations - the following average results were obtained:

		%	%
Solution without error or error controlled	The discriminant method works correctly	76,0%	89,6%
	Error with manual validation	13,6%	
Uncontrolled error	Uncontrolled error	10,4%	10,4%

Table 4.1.: Simulation results

The discriminant method works correctly in two situations:

- When there exist two populations and the intervention of a technician is mandatory to resolve the valuation.

- When there exists only one population and the automatic valuation is correct.

It is possible to apply quality improvement measures such as altering the logistics cutoff value in order to minimize the uncontrolled error or introducing new logic rules. However, these measures will depend on the policies of control of each user.

Applying the model output on a standard city behavior, where valuations in uniform spaces range between 80% and 95%, the results could be:

	Valuations in uniform spaces		
	80%	90%	95%
The discriminant method works correctly	77,6%	78,1%	78,5%
Error with manual validation	17,0%	19,2%	20,2%
Uncontrolled error	5,4%	2,7%	1,3%

Table 4.2.: Quality of the estimate based on the type of city

## 5. Algorithm for calculating the appraised value.

From the set of tools and methods presented in the previous chapter, there evolved a specific calculation algorithm for performing the estimation of the appraised value based on comparative market method. The steps followed are as indicated here:

- i. **Sample selection:** All samples that overcame the restrictions set forth in section 2.1., 2.2. 2.3. were selected.
- ii. **Physical homogenization and sample selection:** Taking the physical characteristics of the object and the homogenizations indicated in section 2.4. it is possible to calculate the homogenized value of the sample and to apply the filters based on the homogenization of Table 2.2. Once these steps are performed by the system, the samples are considered to be "identical" to the object to be valuated.
- iii. **Spatial homogenization:** is performed on the full set of samples previously homogenized and considered as comparable. An estimation is conducted for each subset of 6 samples without repeating them and for each of the spatial estimation methods described in Barris, J. (2008).
- iv. The method showing better **empirical density function** properties will be selected.
  - If the empirical density function generated is considered valid then the midpoint value of the modal interval of the empirical distribution function will be selected as appraised value.
  - If none of the empirical functions meet the properties to be considered valid then the technician will need to resolve the process by choosing the correct modal interval.

## 6. Conclusions

The resulting expert system is clearly a hybrid AVM, as it eventually requires manual intervention. It is a high quality solution that can provide substantial assistance to those officials responsible for conducting the mass valuation processes required by BASEL II and III. It can also be employed as an auditing tool for legal frameworks like BASEL II and III and also for the everyday valuation processes performed within a company when the results of the valuation arise from different providers.

The existence of a diversity of organizations and mechanisms in Europe dealing with individual mortgage valuation processes and mass valuation processes under BASEL II and III, is a difficulty to overcome in order to achieve a homogenized procedure.

In Europe, there are basically two trends in the field of mortgage valuations that derive from the internal real estate market behavior.

Whilst countries like Spain with large valuation companies with high capacity to undertake the valuation of large portfolios and thus to provide the banks with results based on technical processes that are regulated, guaranteed and independent; other countries - due to a different development of their valuation markets - have not produced large companies in this field and have based their procedures on existing internal calculation processes of their banks or are employing generic consultant companies with limited expertise and infrastructure to conduct analysis of the total portfolios.

It is for this reason that we believe that these Expert Systems have the potential to become a homogenization tool of all existing procedures in the European Community. This is the type of tools that an area as sensitive as the mortgage guarantees market requires.

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