Real Estate Investment in an Asset/Liability Modelling Context

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Abstract

This paper examines the role of real estate in pension-plan asset liability modelling. As such, it builds on and extends work in that field by Chun et al (2000) and other work in the financial economics and actuarial literature. Rather than using empirical liability returns, this paper builds a representative liability model of a pension fund, in order to find the impact of changing liability structures on asset allocation. We also examine the impact of expanding the opportunity set of asset classes beyond that used in most earlier work. We find that efficient portfolios in the pension-plan context are quite different from those which are efficient in an asset-only framework. We also find that the liability structure has a significant impact on the efficient portfolios. Furthermore, the introduction of index-linked bonds in the UK changes the efficient portfolios in the immature plan. This may have implications for those countries (such as the US) which are beginning to develop such markets.

1. Introduction

The work of Chun et al (2000) looks at the role of real estate in a pension fund using an asset/liability modelling framework. This follows on from similar work in the UK literature (for example, Ashurst et al, 1998) and work which examines the role of real estate in the context of a mean-variance framework (for example, Firstenberg, Ross and Zisler, 1988). As Chun et al find, if portfolio selection models take into account pension plan liabilities, they can give significantly different results from mean-variance, asset-only models. This paper looks at two aspects of this problem. First, it considers whether the asset/liability modelling approach is the appropriate way of modelling risk in pension fund portfolios. We then critique the approach of Chun et al (2000) and see if further insights can be obtained, particularly from using different pension plan liability structures and a wider opportunity set of assets.

In section 2, we discuss the validity of asset/liability modelling for pension funds. In section 3, we present the liability model. In section 4, we analyse the asset return data. In section 5 we introduce the optimisation procedure used in sections 6-11. In sections 6 and 7 we perform the portfolio analysis and analyse the results in an asset only framework. In sections 8 and 9 we analyse the asset/liability modelling data and derive the appropriate objective function. In sections 10 and 11 we analyse the results of the asset/liability modelling. In section 12 we draw our conclusions.

We find that the optimal portfolios in an asset-only context are quite different from those we find from asset/liability modelling in a pension-plan framework. We also find that the optimal portfolios are dependent upon the liability structure of the scheme. For immature schemes, in particular, we find that the introduction of index-linked bonds into the opportunity set reduces the optimal holdings of real estate.

2. Is Asset/liability Modelling Valid?

The corporate finance view of pension fund asset allocation would reject asset/liability modelling as a valid approach to asset allocation. The corporate finance view regards the pension fund as an extension of the corporation's balance sheet. Asset allocation within the pension fund must therefore be seen in the context of the assets and liabilities of the company as a whole. This often leads to the conclusion that pension funds should only hold bonds because they are more tax efficient, in most jurisdictions, when held in a pension fund. A company which prefers equity assets in the pension fund could make a pure economic gain by repurchasing equity on its own corporate balance sheet and issuing bonds and exchanging equity for bonds within the pension fund. If the liabilities of the pension fund are not dependent on the assets held, there is a pure tax gain from this transaction. The overall gearing of the company, seen as an entity, incorporating its pension fund, remains the same: more bonds are held as assets in the pension fund, matched by an increase in bond liabilities on the corporation's own balance sheet.

There are certainly insights to be gained form this analysis. It could be argued that insufficient attention is given, in the UK in particular, to the overall tax efficiency of the methods by which corporations are financed and to the relationship between the pension fund and the balance sheet of the corporation. However, if this corporate finance view were correct, it would render invalid the practical asset/liability modelling work undertaken by actuaries and also the academic work in this area undertaken in the finance field (for example, by Sharpe and Tint, 1990 and Leibowitz, Kogelman and Bader, 1994), in the actuarial field (by Sherris, 1992 and Wilkie, 1985) and in the real estate field (by Chun et al, 2000 and Ashurst et al, 1998).

There are a number of reasons why the pure corporate finance view may not hold. *Inter alia,* these are:

1. There is sometimes an element of co-insurance in pension funds so that, if the investments fail to perform as expected, discretionary benefits can be reduced.
2. In the UK, pension funds are operated under trust. The trustees take investment decisions in "the best (financial) interest of members" and the balance sheet of the pension fund is not, in a legal or direct financial sense, connected with the balance sheet of the sponsoring corporation. However, it should be mentioned that, if the company is to continue as a going concern, it will generally receive the benefits of any over funding and will have to meet many of the costs of under funding the pension scheme. This is recognised by the new international accounting standards for accounting for pension costs (International Accounting Standard 19).

3. There will be other legal and regulatory constraints on the behaviour of pension funds. These may be explicit (for example investment controls) or implicit, such as the minimum funding requirement or its successor (MFR) in the UK. The MFR requires a valuation of the fund every three years. If the valuation standard is not met, the sponsoring company may have to make cash injections into the fund. Thus there are potential liquidity problems for the sponsoring company if the MFR standard is not met. Ashurst et al (1998) specifically considered this issue. The MFR will be abolished in the UK but it is likely to be replaced by some other statutory test of or disclosure of funding position. The existence of the pension fund guarantee corporation in the US, which sets insurance premiums for pension funds in return for an insurance guarantee of the benefits, can have a similar impact.
4. Regulations surrounding the distribution of surplus of a pension scheme also prevent the pension fund from becoming an extension of the corporate balance sheet. Thus the point made in 2 above should be qualified in that pension scheme surpluses cannot always easily be returned to the sponsoring company.

For the above reasons, we would expect pension fund managers, trustees and advisers to be interested in the impact of investment policy on the expected surplus and variability of surplus[1] of the scheme. As we have indicated above, the surplus of the scheme impacts on the funding cost to the sponsor, on corporate liquidity and on the benefits offered by the scheme, given the ability of the trustees to offer discretionary increases (and sometimes make decreases) in benefits.

Thus it is clear that there is justification for the asset/liability management approach as used by Chun et al. In this paper, we consider the application of their approach to the UK. The paper incorporates a wider opportunity set of assets than were used in Chun et al. We also investigate the impact that the liability structure of pension schemes has on the optimal asset allocation.

3. The Pension Plan Liability Model

The pension fund liability model used in this paper is quite different from that used by Chun et al. Chun et al used data provided by the Business Information file of Compustat to find the mean return (or rate of increase) in the liabilities, the variance of the liabilities and the relationship between asset and liabilities for the pension funds of different industry sectors. In this paper, a different approach is taken for two reasons. The first reason is that the data used by Chun et al are not available in the UK context. The second reason is that a different approach enables us to determine how sensitive the results are to changes in the liability model. Although it is clear that the different industry examples used by Chun et al must have different liability structures, it is not clear what the liability structures were for particular industries, as that information does not form part of the published data. Constructing a

[1] Where surplus is measured as the difference between the actuarial value of the assets and liabilities.

representative model of pension fund liabilities allows us to consider how the role of real estate may change as the liability structure of a pension fund changes.

To investigate this, we construct the liability model in two parts. The first part is the liability model for the mature part of the scheme (members who have already retired). The second part of the liability model is for those members who have not yet retired and are still active in the scheme[2]. In this study, when investigating the impact of the liability structure, we assume that the scheme is either 100% mature or 100% immature. Some schemes will follow one of these structures. In practice, most will fall in between. Nevertheless, it is useful to look at the impact of the extreme cases on optimal investment policy. A more detailed appraisal of the extent to which optimal asset allocation varies with liability structure is the subject of ongoing work.

Equal numbers of individuals are assumed to be members in each age group. For simplicity, we assume an entry into the scheme every five years. Individuals accrue a pension of 1/60 of final salary for each year of their membership, which is assumed to be between age 25 and their current age. The value of the accrued liabilities is determined by projecting salary increases until retirement and discounting the annuity benefits that would be given, based on service to date. The valuation of these liabilities, in respect of the active, working population is carried out with all variables being expressed in "real" terms. The real rate of interest for discounting liabilities before retirement is the long-term rate of return from UK government index-linked bonds. In fact, this choice of valuation rate of interest does pose some interesting methodological problems. In general, UK and US valuation techniques differ. Many although not all, US actuaries would agree that the appropriate rate of interest for valuation would be high quality corporate bond yields. In the UK, where there has been an index-linked bond market for longer than in the US, explicitly index-linked liabilities may be discounted at real rates of interest from index-linked gilts; fixed nominal liabilities would be discounted at a conventional gilt yield and salary-linked liabilities often discounted at the expected rates of return from equities. Accounting standards in the US (for example, FAS 87) require discounting for all liabilities at good quality corporate bond yields. This has also been proposed for use in the UK (FRED 20), for the purposes of determining future pension liabilities on corporate balance sheets. There is, nevertheless, a good case for using the rates of return from index-linked bonds to discount both index-linked liabilities and salary-linked liabilities (and, indeed, this may be allowed if FRED 20 is applied in the UK). This means that no inflation projection needs to be carried out as all variables are defined in real terms. After retirement, we have assumed that liabilities are fixed in nominal terms, discounting is carried out at the yield from conventional gilts.

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[2] We ignore members who have left the scheme and have "frozen" benefits.

The real rate of salary increases for projections is assumed to be 3%[3]. The value of the liabilities can thus change for two reasons. Interest rates can change and actual salary increases can be different from projections.

The second part of the liability model represents the liabilities in respect of those members who have already retired. This part of the scheme will also have pensioners at representative five-year age intervals. Pensions in payment are assumed to be fixed nominal, amounts[4]. Specifically, the pensions in payment liabilities are determined as follows: it is assumed that, at age 65, an equal number of people had retired in each of the past 35 years (again we take representative ages of 70, 75, 80 and so on); they each had a pension of £18,579 (the pension accrued by 65 year olds in the active part of the scheme) reduced by an assumed rate of average real earnings growth in the period since retirement for each year of age after 65 (as this represents the period since retirement). It is therefore implicitly assumed that historically, pensions in payment have been uprated to compensate for inflation but that, in the future, there will be no such uprating. Again, this is common in UK schemes, where uprating is discretionary. It may be assumed that such uprating will not occur but, if there are surpluses within the scheme, they may be used to provide price indexation (see Faculty and Institute of Actuaries, 1999). Mortality tables are then used to determine the proportion still living at each age and future payments are then projected using the mortality tables and discounted. The mature liabilities are expressed in nominal terms and discounted at nominal rates of interest from long-term conventional government bonds.

4. Analysis of the Asset Return Data

The data period is limited to the length of the return series for index-linked gilts. We have started the data series in 1984, by which time index-linked gilts had been in existence for nearly three years and had become reasonably marketable. The matrices for expected returns, standard deviations of returns and covariances of annual returns from assets are estimated from historical data over the period 1984 to 2000 inclusive[5]. Mean returns and standard deviation of returns are shown in table 4.1.

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[3] This is equal to the average rate of real salary growth over the recent past plus an assumed scale rise of 0.5% per annum.

[4] In the UK, there is considerable variation in terms of the way in which pensions in payment are varied. Some will be fixed in nominal terms; some will be fixed in nominal terms but trustees will give discretionary uplifts if there is a surplus in the fund; some will be fixed in real terms; and some will be subject to “limited price indexation” so that they change with changes in the price level with a minimum increase of 0% and a maximum of (say) 3% or 5%. We do not model these particular circumstances. However, such modelling may provide an interesting extension of this work. In particular, the limited price indexation liabilities have an option structure which has similarities with (but is still different from) the upward only rent review option structure in the traditional UK institutional lease.

[5] By way of comparison, Chun et al used a data period from 1989 to 1997, in their case constrained by the limited availability of liability data.

Asset returns were taken from the Barclays Capital (2001); exchange rates were taken from the Bank of England Statistical Abstract (2001 and earlier volumes); real estate return figures were taken from IPD (2001).

The ranking of the mean return figures is generally consistent with a priori expectations. It is worth noting that the mean return from conventional gilts is slightly higher than that from real estate. The period as a whole was one in which inflation expectations fell significantly. It is therefore not surprising to see relative out performance of a long-term nominal asset. The ranking of the standard deviation of return figures is also intuitive, although this requires more explanation. US equity returns are more volatile than UK equity returns largely as a result of the added impact of exchange rate volatility (all returns are converted into sterling terms). The standard deviation of dollar US equity returns is 14.5% and the standard deviation of the return on £1 invested in dollars is 11.4%. Thus the standard deviation of dollar US equity returns is only slightly higher than that from sterling UK equity returns. The standard deviation of conventional gilt returns is higher than that from real estate returns. Whilst this may seem surprising, the period 1984-2000 was a period of significant volatility in expected inflation. The standard deviation of cash returns, unsurprisingly, is the lowest of all asset classes. The standard deviation of returns from index-linked gilts is lower than that from conventional gilts. This can be explained by the fact that, whilst index-linked gilts of a given term to redemption have a longer duration than conventional gilts, the values of index-linked gilts are only affected by changes in real rather than in nominal yields.

Asset class	**Mean return %**	**Standard deviation of return %**
UK Equities	15.6	12.6
US Equities	16.9	19.8
Real estate	10.9	9.5
Index-linked gilts	8.7	7.3
Cash	9.1	3.1
Conventional gilts	11.6	9.7

Table 4.1 Mean and standard deviation of asset returns

Portfolio decisions are, of course, taken on the basis of ex-ante expectations and not ex-post values of mean and standard deviation of return. It is therefore reasonable to ask, before proceeding to determine efficient portfolios, whether the asset return statistics would form a reasonable basis for expectations. In terms of the ordering of mean returns and of standard deviations of returns, the data are not unreasonable. However, in terms of the magnitude of real returns, relative to equity returns, it can be said that index-linked gilts have produced real returns that may seem unreasonably low. The effective equity risk premium over index-linked bonds, at 6.9%, is high. Whilst much work has been undertaken on the equity risk premium, the index-linked bond market has not been in existence for long enough for this work to consider the risk premium in relation to that asset class. Nevertheless, the magnitude of the risk premium is not so unreasonable that it should not be used as a basis for the simulations in this paper. It should also be mentioned that all the real return figures are historically high. However, in asset/liability studies, it is the relative returns and the correlation structure (including the correlations between the assets and liabilities) which are most important.

Indeed, for the relevant asset classes, the mean and standard deviation of return structure is very similar to the statistics for the data used by Chun et al (if the data for the two domestic markets, UK equities in our case and US equities in the case of Chun et al are compared). The major difference is in the returns from real estate. However, here Chun et al used the returns from real estate investment trusts (REITs), rather than from direct real estate. Unsurprisingly, Chun et al found a higher mean and higher standard deviation of return.

The correlation matrix for the asset classes is shown in table 4.2.

Correlation matrix	**UK equities**	**US equities**	**Real estate**	**IL gilts**	**cash**
UK equities	1.00				
US equities	0.81	1.00			
Real estate	0.17	-0.05	1.00		
Index-linked gilts	0.49	0.47	0.21	1.00	
Cash	0.01	-0.04	-0.30	0.00	1.00
Conventional gilts	0.34	0.26	0.02	0.76	-0.01

Table 4.2 Asset correlation matrix

There are no particularly surprising results in the correlation matrix. Real estate has a low correlation with other asset classes. This may be due to the use of valuation-based data to compile the indices from which real estate returns were derived. The extent to which the low correlation between real estate and other asset classes is explained by the use of valuation-based indices is the subject of a substantial literature and further work on the impact of such data on asset allocation decisions is being undertaken by the author. The correlation between US equity returns and UK equity returns is higher than is quoted in other studies. However, it should be remembered that the US equity returns are in sterling terms. The correlation coefficient between UK equity returns and US equity returns both expressed in their domestic currencies is 0.66. But there is a positive correlation coefficient of 0.51 between UK equity returns and the return from investing sterling in dollars[6]. This may be a surprising result. However, it is compatible with both purchasing power parity theory and the Fisher hypothesis. As expected, index-linked and conventional gilt returns have a high correlation (both are affected by changes in real credit-risk-free yields). Cash has a low or negative correlation with all asset classes. Again, this is not surprising, as high cash returns will tend to imply a tight monetary policy which, in turn, may impact

[6] This implies a negative correlation between the sterling:dollar exchange rate (expressed S/£) and UK equity returns.

adversely on returns from other asset classes due to its short-term impact on the real economy. It is notable that real estate returns were particularly low during the period of tight monetary policy and high cash returns in the early 1990s.

Chun et al found a higher correlation between conventional bonds and equities. This may be due to the lower fluctuations in expected inflation in the US than in the UK. Further evidence for this explanation is provided by the fact the index-linked gilts have returns that are more highly correlated with UK equities than conventional gilts. Chun et al also found that real estate was more highly correlated with both bonds and equities than is the case here. Again, that may be due to their use of REITs' returns as a proxy for property return data.

5. Optimisation

There are three major differences between our approach and the Chun et al approach. In our work, we use an overseas equity class (US equities) rather than just domestic equities. Chun et al use only US investment classes. In addition we also use index-linked gilts. The third difference is the different approach to liability modelling (see below).

In order to find the impact of widening the asset opportunity set on efficient portfolios, we perform our analyses in two stages. First, we restrict portfolios to cash, UK equities, long-dated bonds and real estate, which provides the closest comparison with the modelling of Chun et al. We will then add in the additional asset classes used in our study. This will help us understand whether optimisation produces different results when the additional asset classes of index-linked gilts and overseas equities are introduced. In particular, it will help us understand whether the removal of exchange controls in 1979 and the subsequent introduction of index-linked gilts in 1981 can explain the decline in the allocation of UK investment fund assets to real estate. These results relating to the inclusion of index-linked gilts in pension-plan optimisation may be of interest in the US, given the relatively recent development of the index-linked bond market in that country.

6. Optimisation in an Asset-only Framework

We look at a number of efficient portfolios, with different levels of mean and standard deviation of return, ignoring pension-plan liabilities. The asset classes are first restricted to UK equities, cash, long-dated government bonds and direct real estate and the minimum standard deviation of return portfolio is found for four levels of expected return. In addition, we find the minimum variance and maximum expected returns portfolios with and without restrictions on the number of asset classes. The restrictions were then removed, so that efficient portfolios could include US equities and index-linked bonds. In fact, in the asset-only example, in no case was either the standard deviation of return or the expected return of an efficient portfolio changed (to one decimal place) by the removal of restrictions. The efficient portfolios we found are shown in the table 6.1.

Asset class	Proportion in asset class %									
A= without restrictions B = with restrictions	A	B	A	B	A	B	A	B	A	B
Risk level	Minimum risk		Low risk		Mid risk		High risk		Very High Risk	
UK equities	0	0.2	8.6	13.7	31.8	35.3	62.7	64.1	100	100
US equities	1.7	-	3.9	-	2.7	-	1.0	-	0	-
Real estate	14.6	14.5	16.6	15.2	17.3	16.4	18.2	17.9	0	0
Index-linked gilts	0	-	0	-	0	-	0	-	0	-
Cash	77.9	78.6	61.9	62.1	35.6	35.8	0.6	0.6	0	0
Conventional gilts	5.8	6.7	9.0	9.0	12.6	12.6	17.5	17.4	0	0
Expected return %	9.66	9.56	10.5	10.5	12.0	12.0	14.0	14.0	15.64	15.64
Standard deviation of return %	2.48	2.50	3.10	3.13	5.48	5.49	9.23	9.23	12.64	12.64

Table 6.1 Efficient portfolios in the asset only framework

7. Analysis of the Efficient Portfolios

It is notable that extending the opportunity set to include US equities and index-linked bonds has virtually no impact on the efficient portfolios in the asset-only framework. The absence of index-linked bonds can be explained by their low mean return (not compensated by a significantly lower standard deviation of return than conventional bonds and property). They also have a high correlation with UK equities and conventional bonds. Intrinsically, index-linked bonds are not low-risk assets in the one-year horizon, nominal return, asset-only model. They are also not high-return assets. Unsurprisingly, they appear neither in the low-risk, nor high-return portfolios. There will be an interesting comparison to be made here with the results relating to pension plan portfolios, where the liabilities are of longer duration and correlated with inflation. The main reason for the absence of US equities from the unconstrained portfolios would appear to be the high correlation of US equity returns (converted into sterling) with UK equities. If the correlation between sterling UK equity returns and dollar US equity returns is used (0.62) the proportion of US equities in the high-risk, unconstrained portfolio increases to 12%. If the dollar return figures are also used for standard deviation of return and expected returns from US equities, the proportion of the high risk portfolio that is invested in US equities rises to 33%. The extent to which the high correlation between US and UK equity sterling returns arises as a result of the international diversification of the activities of UK companies has not been investigated. Nevertheless, our results may provide some prima facie support for the evidence that Errunza et al (1998) provide for gains from diversification being achieved without investment abroad. Clearly, the low-risk portfolios include considerable cash proportions. Again, this is not surprising in the one-period, nominal return model.

In fact, the efficient portfolios whether constrained or unconstrained are remarkably similar to those of Chun et al. The real estate proportion is between 10% and 20% in both studies. In Chun et al's study, intermediate government bonds (the nearest equivalent to cash in our model) dominate the low-risk portfolios. Domestic equities dominate the high-risk portfolios in both studies. The only significant difference between the results of Chun et al and our results is that long-dated government bonds do not appear in any of the Chun et al efficient portfolios, whereas they do appear in our study. This may be explained by the fact that, in our study, the mean long-dated government bond return is closer to the mean domestic equity return than in Chun et al's study. Also, the correlation between domestic equities and bonds is higher in Chun et al's study than in our study. Both of these features can be explained by the importance of falling inflationary expectations in determining the structure of UK government bond returns.

The four main conclusions from this section are:

?? Real-estate has an important role to play in diversified, asset-only portfolios
?? The results found for the US, by Chun et al (2000) are not dissimilar to those found for the UK when a similar methodology is applied

- ?? The exclusion by Chun et al of two important asset categories does not appear to be important in the asset-only framework
- ?? The different structure of long-dated bond returns in the UK makes them part of the efficient portfolios

8. Analysis of the Asset/liability Modelling Data

The mean rate of growth of the liabilities of the mature scheme is 10.56%. The standard deviation of the rate of growth of the mature liabilities is 5.93%. The mean rate of growth of the liabilities of the mature scheme is 11.42% and the standard deviation of the rate of growth 11.62%. The correlation structure between the pension plan liabilities and the asset classes is shown in table 8.1.

Correlation matrix	UK equities	US equities	Real estate	IL gilts	Cash	Conv gilts
Liability mature scheme	0.27	0.14	0.11	0.72	0.04	0.97
Liability immature scheme	0.28	0.25	0.23	0.88	-0.22	0.91

Table 8.1 Asset and liability correlation coefficients

These statistics are intuitively reasonable. The mature liabilities involve fixed annuity payments and should therefore behave in a similar way to conventional gilts. They have a similar mean return and the correlation between the mature liabilities and conventional gilts is very high. The standard deviation of the rate of growth of the mature liabilities is less than the standard deviation of the long-gilt portfolio because the duration of the payments is lower. A priori, these statistics would lead us to expect that low risk portfolios would be a duration-matched mix of cash and long-dated gilts.

The liabilities of the immature scheme have stronger correlations with the real[7] asset classes than do the liabilities of the mature scheme. The correlation with conventional gilts is lower. A priori, we would expect to see greater representation of index-linked gilts in the low-risk portfolios and greater representation of US equities in the high-risk portfolios. The standard deviation of the immature liabilities is greater than that of the mature liabilities. The reason for this is that the immature liabilities depend on a complex interaction of inflation, real bond yields, nominal bond yields and real wage growth. The liabilities of the mature scheme are mainly dependent on bond yields.

9. Derivation of the Objective Function for Asset/liability Modelling

[7] We refer to asset classes which, on *a priori* theoretical grounds, we might expect to exhibit the characteristic of hedging unanticipated changes in inflation.

Various different objective variables and objective functions have been proposed for pension plan asset/liability modelling. Wilkie (1985) and Sherris (1992) effectively used absolute levels of surplus. In the Wilkie formulation, three-dimensional efficient frontiers were developed where the asset allocation decision depended on the mean and standard deviation of the surplus and the "price" of the portfolio. Booth It can be shown that this is equivalent to using mean and standard deviation of surplus and the initial level of surplus as objective variables. Thus, for example, the plan sponsor could choose a higher initial level of surplus (by purchasing more assets) and, as a result, have a higher expected surplus for a given portfolio. Focusing on the absolute level of surplus of a pension scheme is helpful when performing asset/liability modelling for particular schemes. However, it is less useful for more general theoretical and empirical work. It is difficult to develop appropriate objective functions based on the absolute level of surplus. Unlike wealth, in consumer-choice utility theory, surplus is not an absolute quantity, but merely forms part of the overall wealth of the scheme's beneficiaries or plan sponsors.

An alternative would be to use "surplus return" as the objective variable. This could be defined in a similar way to the return on an asset portfolio (i.e. $SR_t = (S_t/S_{t-1}) - 1$ where S_t is the surplus at time t and SR_t is the surplus return in year t). However, whereas the return on an asset portfolio gives rise to the same rate of increase in invested wealth, whatever the starting value of wealth, so that it is possible to derive tractable portfolio selection conclusions using utility theory, this not the case with "surplus return". For example, a 10% increase in surplus or 10% surplus return will give rise to a different increase in plan sponsor and plan member wealth if the starting surplus were \$100m, compared with if it were \$1m. Surplus return can also produce infinite or undefined values when the starting or finishing level of surplus is non-positive. Chun et al describe their objective function as "surplus return". However, if we define surplus return as SR_t above, Chun et al in fact use $SR_t*(S_{t-1}/A_{t-1})=(S_t-S_{t-1})/A_{t-1}$. Thus they use surplus return standardised by the starting level of assets in the scheme or the increase in surplus expressed per unit initial assets. Such a measure, or a similar measure standardised by the starting level of liabilities, would seem an appropriate objective function. This is because surplus can be used to increase member's benefits (plan liabilities) or to return assets to the employer either directly or by reducing future contributions. It is therefore reasonable to focus on the amount of any increase in surplus expressed per unit initial assets (or liabilities). We take that approach in this paper.

An alternative was proposed by Leibowitz et al (1994) who used "funding ratio return" (effectively the rate of increase of the funding ratio or in the ratio of assets to liabilities). As this is the difference between two ratios divided by a ratio, it can give rise to results which are difficult to interpret. For example, if there is a 10% increase in the funding ratio when it is at a level of 1.1, this will imply a smaller increase in wealth than if the initial funding ratio were (say) 2. Leibowitz et al recognise this and suggest that, as a result, risk tolerance would have to vary with the initial level of surplus, for a given scheme. This varying risk tolerance would have to handle an additional problem. Not only would similar funding ratio returns imply different changes in plan sponsor and member wealth at different starting levels of funding ratio, plan sponsors are likely to be more averse to a fall in the funding ratio, the smaller its starting level. This latter problem would also arise with the approach we take in this paper. However, we will look at one representative scheme with no initial surplus (that is present value of assets is equal to present value of liabilities).

We will use the following notation:

SR'_t = surplus return, standardised for initial asset values, in year t
A_{t-1} = assets invested in the scheme at time t-1
x_i = proportion of asset portfolio invested in asset i
R_i = rate of return on asset i
R_l = rate of return (rate of increase) in the plan's liabilities
L_{t-1}= scheme liabilities at time t-1

$$SR'_t \, ? \, \underset{i?1}{\overset{i?n}{?}} x_i R_i \, ? \, R_l \, ? \, \frac{L_{t?1}}{A_{t?1}}$$

This is essentially the same as the Chun et al objective variable, although the notation is different. The standardised surplus return variable can be regarded as the rate of return on the asset portfolio less the adjusted rate of return on the liabilities. The adjustment is the ratio of liabilities to assets at the beginning of the period. It can easily be shown that this is equivalent to the increase in surplus per unit initial assets. x_i*s* are the control variables (although it may also be possible to control R_l to some extent if there is any discretion with regard to benefits paid by the scheme). Appropriate values for the x_i*s* will be chosen to maximise the objective function relating to SR'_t.

We assume that pension plans maximise the objective function:

$$F(SR'_t) = E(SR'_t) - a{*}?(SR'_t)^2$$

where:

$$E(SR'_t) \, ? \, \underset{i?1}{\overset{i?n}{?}} x_i E(R_i) \, ? \, E(R_l) \, ? \, \frac{L_{t?1}}{A_{t?1}}$$

with
$E(R_i)$ = expected rate of return from asset *i*
$E(R_l)$ = expected rate of growth (rate of return) of the liability

and

$$? \, (SR'_t)^2 \; ? \underset{i?1}{\overset{i?n}{?}} x_i ?_i^{\,2} \; ? \underset{i?1}{\overset{1?n}{?}} \underset{j?1}{\overset{j?n}{?}} ?_{ij} x_i x_j \; ? \, \frac{L^2_{t?1}}{A^2_{t?1}} \, ? \, ?_l^{\,2} \; ? \, \frac{L^2_{t?1}}{A^2_{t?1}} \, ? \underset{i?1}{\overset{1?n}{?}} ?_{il} x_i \; ? \, \frac{L_{t?1}}{A_{t?1}}$$

with
?$_{i}$ = standard deviation of return from asset *i*
?$_{ij}$ = covariance of return between asset *i* and asset *j*
?$_{l}$ = standard deviation of return of the liability
?$_{il}$ = covariance of return between asset *i* and the liability.

Our objective function is similar to that of Chun et al except that *a=0.5*?.* Clearly the asset portfolio which maximises the objective function used here will maximise the objective function of Chun et al, as long as *a* and ? are chosen appropriately. The form used in Chun et al is familiar in financial economics because ? is the risk tolerance parameter, as defined by Pratt (1964) and used in pension plan modelling by Sharpe and Tint (1990). However, this author regards it as confusing to use ? in this context, as it would not have the same precise meaning, defined by Pratt.

Just as ? can be regarded as a risk tolerance parameter, *a* can be regarded as a risk aversion parameter. The higher is *a*, the greater will be the pension plan's aversion to risk. If *a* = 0, the pension plan sponsors will be risk neutral and will try to maximise expected surplus. As *a*? ?, the plan sponsor will choose the portfolio that will minimise the variance of the plan surplus.

10. Optimisation for the Mature Scheme

As in the asset-only case, we determined efficient portfolios without and with index-linked gilts and US equities (constrained and unconstrained). We found portfolios which maximised the values of the objective function, defined above, for sample values of *a* between 0.02 and 1). We also found global minimum variance portfolios and the portfolio which maximised returns with and without constraints. In the final two columns of table 10.1 we found the maximum expected return portfolio with constraints (100% UK equities) and then found the portfolio without constraints which provided the same expected return. The efficient portfolios are shown in table 10.1.

Asset class	Proportion in asset class %									
A= without restrictions B=with restrictions	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
Risk level	**Minimum risk**		**a=1**		**a=0.15**		**a=0.02**		**Very High Risk**	
UK equities	0	0	0	0	9.6	9.6	69.2	69.2	96.9	100
US equities	0	-	0	-	0	-	0	-	2.3	-
Real estate	9.9	9.9	10.4	10.4	11.1	11.1	0	0	0	0
Index-linked gilts	0	-	0	-	0	-	0	-	0	-
Cash	29.4	29.4	27.7	27.7	15.5	15.5	0	0	0	0
Conventional gilts	60.7	60.7	61.8	61.8	63.8	63.8	30.8	30.8	0.8	0
Expected surplus return	0.26	0.26	0.29	0.29	0.97	0.97	3.80	3.80	5.01	5.01
Standard deviation of surplus return	1.20	1.20	1.21	1.21	2.03	2.03	8.48	8.48	12.20	12.20

Table 10.1 Efficient portfolios for the mature pension plan

The low-risk portfolios contain a mix of conventional gilts and cash, reflecting the liability profile of the mature scheme. The proportion of real estate in the portfolios is remarkably stable at around 10%, except for the high-risk portfolios, in which real estate does not feature. Extending the opportunity set of assets to include index-linked gilts and US equities has virtually no impact on the efficient portfolios, except that the maximum expected return portfolio will, of course, be 100% US equities. Increasing the risk tolerance clearly leads to an increase in the UK equity proportion and a reduction in cash, real estate and conventional gilts. It is of interest to consider the practical viability of the portfolios of different levels of risk. If we consider a pension scheme which has zero initial surplus. The trustees may be interested in ensuring that the funding ratio does not fall below 95%[8] with a given probability (say 5%). This is equivalent to finding the probability of a surplus return of –5%, in the case of our model scheme. It will be assumed that the surplus return is normally distributed. The efficient portfolio for a = 0.02 provides a probability of falling below a 95% funding level of 14.9%; the efficient portfolio for a = 1.5 provides a probability of falling below a 95% funding level of 0.16%. These are probably higher than trustee would like and lower than trustee would like respectively. If a = 0.05, the probability that the funding ratio does not fall below 95% is 5.26%: this might be a reasonable position. The portfolio which maximises the objective function at that value of a is 31.6% UK equities, 6.8% real estate and 61.6% UK gilts. This might be regarded as a reasonably practical portfolio for many trustees managing a closed fund of mature liabilities.

We will not make a comparison with Chun et al or with asset allocation decisions in practice at this stage, because the mature scheme is a particular case with a special liability structure. It is neither close to the average liability structure nor to the liability structure modelled by Chun et al. Nevertheless, whilst little published data on mature scheme asset allocation is available, this is an interesting special case, as many schemes are in the position of being mature either as a result of them being closed to new entrants and replaced by defined contribution schemes or because of the maturing of the workforce in a declining industry.

11. Optimisation for the Immature Scheme

As in the mature-scheme case, we determined efficient portfolios without and with index-linked gilts and US equities (constrained and unconstrained). We found portfolios which maximised the values of the objective function, defined above, for sample values of a between 0.02 and 1). We also found global minimum variance portfolios and the portfolio which maximised returns with and without constraints. The efficient portfolios are shown in table 11.1. The portfolios under the “very high risk” column have the same risk and return characteristics as the equivalent portfolios in table 10.1.

[8] This is particularly the case in the UK, due to various aspects of pension fund legislation.

Asset class	Proportion in asset class %									
A= without restrictions B=with restrictions	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**	**A**	**B**
Risk level	**Minimum risk**		**a=1**		**a=0.15**		**a=0.02**		**Very High Risk**	
UK equities	0	0	0	0	0	3.7	42.2	56.1	76.1	100
US equities	0	-	0	-	3.7	-	11.6	-	18.9	-
Real estate	0	8.1	0.3	7.9	6.0	5.2	0	0	0	0
Index-linked gilts	30.1	-	26.0	-	0	-	0	-	0	-
Cash	0	0	0	0	0	0	0	0	0	0
Conventional gilts	69.9	91.9	73.7	92.1	90.2	91.1	46.2	43.9	4.9	0
Expected surplus return	-0.68	0.14	-0.56	0.14	0.35	0.31	2.48	2.41	4.2	4.14
Standard deviation of surplus return	4.55	4.80	4.56	4.80	4.89	4.90	9.09	9.02	14.17	14.17

Table 11.1 Efficient portfolios for the immature pension plan

The low risk portfolios contain a mixture of index-linked gilts and conventional gilts. This reflects the liability profile of the scheme. The liabilities depend on wage inflation and are discounted for part of their term at expected returns from index-linked bonds. There is no perfect matching combination of assets for the immature liabilities (unlike in the mature case). The liabilities depend on a complex interaction of inflation, real bond yields, nominal bond yields and real wage growth. The composition of the low-risk asset allocation is therefore intuitive. As the risk tolerance increases, there is an increased allocation first to conventional gilts and then to UK and US equities, at the expense of real estate and index-linked gilts. The former trend reflects the fact that whilst there are higher expected returns from conventional bonds compared with index-linked gilts they have poorer inflation hedging characteristics. It is possible that the higher expected returns result is a consequence of the data period used, which was one during which inflation expectations fell and returns from conventional bonds were particularly high. If conventional bond returns are constrained to be 0.5% above index-linked bond returns, which might be a more reasonable estimate of the excess return of conventional over index-linked bonds (it is generally accepted that there should be some inflation risk premium from conventional bonds) the conventional bond allocation falls by 14% in the a = 1, unconstrained scenario.

It is noteworthy that the standard deviation of surplus is significantly higher for all optimal portfolios for the immature scheme than it is for the equivalent portfolio for the mature scheme. There are two reasons for this. The first reason is that the liability structure of the immature scheme is more complex and depends on a greater number of stochastic elements (see above). The second reason is that the immature liability cannot be matched easily by a single asset class or combination of asset classes.

In the case of the immature scheme, imposing the constraints on the asset allocation does significantly affect the optimal allocation. It can be seen that, for low risk portfolios, expanding the opportunity set to include index-linked bonds would lead to a substantial allocation to that class, at the expense of real estate and conventional bonds. At higher levels of risk tolerance, expanding the opportunity set leads to significant allocations to US equities at the expense of UK equities. It is not surprising that the introduction of extra asset classes has more impact on efficient portfolios in this case, given that both additional asset classes are “real” asset classes and the liabilities are partially “real”.

It is notable that the minimum standard deviation of surplus return achievable in the work of Chun et al was 2.38%, whereas in our case it is 4.55%. It is unclear why this might be the case as we have no published information on the liability structure in the underlying schemes from which the data of Chun et al was collected. One can speculate that the greater instability of inflation in the UK may be a cause. The asset allocations in the low risk portfolios constructed by Chun et al are similar to those we have found, where we have constrained the opportunity set (assets being divided between conventional bonds and real estate with something less than 10% allocated to real estate).

It is notable that there is a significant move into index-linked bonds from both real estate and conventional bonds when the opportunity set is widened. This may have lessons for the US, given the recent introduction of index-linked bonds to that market. As is the case for Chun et al, the proportion of assets allocated to real estate reduces sharply as risk tolerance increases (this is less obvious for the unconstrained case where optimal allocations to real estate peak and then tail off as risk tolerance increases). Both Chun et al and our work demonstrate a gradual movement out of bonds and into equities when risk tolerance reaches higher levels. In our case, the equity allocation includes an allocation to US equities. Our high risk portfolios have both lower expected surplus return and higher standard deviation of surplus return than those found by Chun et al.

We considered the probability of having a surplus of less than –5% from different investment strategies (as was also considered for the mature scheme in section 10 above). In fact, there is no investment strategy which will provide a probability of less than 5% of such an outcome. No investment strategy had a probability of surplus return less than –5% which was less than 15%. Again, this is a reflection of the interaction of stochastic factors which determine the liabilities of the scheme and the difficulty of matching the liabilities. It is clear that, as lower return portfolios are sought, the reduction in expected surplus return contributes to increasing the probability of falling short of the target by nearly as much as the decrease in the standard deviation of return contributes to decreasing the probability of falling short of the target. This may explain the characteristic of UK pension funds investing heavily in asset categories which appear risky in an asset-only framework.

12. Conclusions

It is clear that asset/liability modelling within a pension plan produces optimal portfolios which are significantly different from those found in traditional mean-variance asset portfolio optimisation. The particular liability structure of the pension plan also has a significant impact on optimal asset portfolios, particularly at low levels of risk tolerance. In general, low-risk portfolios in a mature plan should be invested in real estate, cash and conventional gilts and low risk portfolios in a mature plan in conventional and index-linked gilts.

Real estate features in all but the highest risk portfolios in mature pension plans and also in the asset-only optimisation. However, this only holds true for the immature pension plan if the asset classes are restricted to exclude index-linked gilts and US equities. Typically, optimal levels of real estate are around 10% in the mature pension plan and between 5% and 10% in an immature plan which exclude the possibility of index-linked bond and overseas equity investment. This has implications for future pension-fund investment policy as pension funds are expected to mature in the UK. It is notable that when the asset opportunity set is extended to include index-linked gilts and US equities, optimal asset allocations hardly change in the case of the asset-only optimisation and the mature pension plan. However, in the immature plan, real estate does not feature significantly in the low risk portfolios, when the asset opportunity set is extended. In mid-risk portfolios, the proportion of real estate increases slightly, however, to 6% from 5.2%. The introduction of index-linked gilts and the removal of exchange controls in 1979 may therefore have been an influence on the reduction in the proportion of real estate held in UK pension plans over the last twenty years. There may be lessons here for the US, where an index-linked bond market is currently developing.

There are many similarities between our results and those of Chun et al, giving confidence in the overall framework. However the approach taken allows extra insights to be gained regarding the evolution of optimal asset allocations as pension fund liability structures change and as new asset markets develop.

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